

Université Panthéon - Assas

École Doctorale d'EcoGeInfoCom (ED 455)

Thèse de doctorat en Sciences Économiques

soutenue le 17 janvier 2014

Thèse de Doctorat / Janvier 2014

Accumulation des Biens, Croissance et Monnaie



Université Panthéon-Assas

Jean Marie Cayemitte

Sous la direction du Professeur Damien Gaumont

Membres du jury :

Rapporteurs :

Bertrand Wigniolle, Professeur à l'Université Panthéon-Sorbonne, Paris I

Julian Revalski, Professeur à l'Académie Bulgare des Sciences, Sofia, Bulgarie

Membres :

Joël Blot, Professeur à l'Université Panthéon-Sorbonne, Paris I

Bertrand Crettez, Professeur à l'Université Panthéon-Assas, Paris II

Damien Gaumont, Professeur à l'Université Panthéon-Assas, Paris II

Avertissement

L'université n'entend donner aucune approbation ni improbation aux opinions émises dans cette thèse ; ces opinions doivent être considérées comme propres à leur auteur.

I dedicate this thesis to my family :

*Specially to my wife Kettleine who has encouraged me to undertake and finish this
manuscript;*

To my children Bradley, Kindersley, Garfield and Lucas.

Acknowledgments

I would like to thank the Center of Research on Markets, Employment and Simulation (ERMES) of Pantheon-Assas University for allowing me the opportunity to undertake my PhD since March 2010. In this regard, I owe a special debt to Professor Damien Gaumont, Director of ERMES, for his important role as supervisor of my PhD dissertation and with whom I had opportunity to work on many important research in economics. I also thank Professor Joel Blot (Pantheon-Sorbonne University) for his contribution to the first theoretical model developed in this thesis and his advice in the mathematical structure.

I thank all the professors and researchers at the ERMES, particularly Professor Bertrand Crettez for his useful advice and for allowing me the opportunity to work with him as part-time lecturer, Professor Jean Mercier-Ethier for his collaboration, Olivier Cardi for our useful discussion. I am grateful to professor Joël Blot, professor Bertrand Crettez, professor Bertrand Wrignolle and professor Julian Revalski (Université des Antilles et de la Guyane(UAG)) for being members of dissertation committee. Thanks to professor Revaski (my supervisor of dissertation in Master of mathematics), with whom I had the opportunity to work for the first time on general equilibrium of Debreu in 2006 at the University of French West Indies (UAG).

I also thank Professor Randall Wright of University of Wisconsin for his advice on OLG version of this thesis and Mr. Louis Bê Duc of Banque de France for our discussion.

I thank the State University of Haiti, particularly the Faculty of Law and Economics (FDSE) and the France Embassy in Haiti for granting this research doctoral thesis. In this regard, I would like to thank professor Fritz Deshommes, Vice-rector of Research for his support in carrying out this thesis. I acknowledge Gelin I. Collot professor and former Dean of FDSE for advising me to undertake this academic research. I am highly indebted to professor Wolf Jovis Bellot, Dean of FDSE, for his useful advice and collaboration. I never forget Mr. Bonnet and Mr. Meus for their support and understanding. I also thank all the professors of FDSE, particularly Eddy Labossière for our discussions on our respective doctoral thesis.

I would like to Thank Ecole Normale Supérieure (ENS) of Port-au-Prince, particularly the board members : Berard Cénatus, Etienne and Rosemond for their patience and understanding.

I thank the Bank of the Republic of Haiti (Central Bank), particularly members of the Board : Charles Castel (Governor) for his significant support and understanding in achieving this doctoral thesis, Georges Henry Fils (Deputy Governor), Jean Baden Dubois (Chief Executive Officer), Georgettes Jean-Louis (member of the Board) and Duroseau (member of the Board). I thank also Philippe Lahens (former deputy Governor) and Remis Montas (former member of the Board) for their support and understanding. I also thank all the members of Money and Economic Analysis Department (MAE), particularly Carlo Janvier (Director), Erol Saint-Louis (Deputy Director), Ronald Gabriel (former Director), Henry Dubois (Director of IFBC). I also thank all the MAE staff for their understanding and good collaboration : Jovis Bellot, Gilles Colbert, Jean Naurable Charles, Jean Lesly Robertson Leger, Julnor George, Nithder Yves Pierre, Ted Augustin, Jemley Jean-Baptist, Carl-Henri Prophète, Augustin Dudley, Ruthland Borga, Sukaina Edouard, Lyssandre Leandre, Debora Descieux, Gihanne Amboise, Claude Jean Divers, Jode Beauplan. I owe a special thank to all MAE secretaries : Mrs. Augustin, Mrs. Cooper and Miss Orizier (former secretary of MAE).

I thank all the PhD students of the ERMES with whom I spent good moments and for being good friends. So I cite particularly Hayet IRZIL, my co-author with whom I have worked on both real options theory and the stocks market, Roger Antoun, Sophie Bienenstock, Inaam Chaabane, Ahmed Chtourou, Nicolas Dumas, Adelaide Fadhuile-Crepy, Emilio Carrera Felix, Farah Hachicha, Narjès Khili, Elie Kolakez, Myriam Kouki, Meriem Maatig, Edwige Marion, Sylvain Mignot, Maïva Ropaul, Stephanie Saba, Aguibou Tall, Emanuel Valat.

I am very grateful to Josette Valentin and Naima Baba-Aïssa for helping me throughout my stay in Paris. With them, I found everything I needed in terms of information about laboratory ERMS and the Panthéon-Assas University.

I would like also to thank all the PhD students of Laboratory LEM, particularly Alassa

Mfouapon with whom I have had good discussions particularly on estimation of the Dynamic Stochastic General Equilibrium (DSGE) models for Cameroon economy and for being a good friend, Maximilian and Lemoyne.

I also thank all those who contributed directly or indirectly to the success of the doctoral thesis.

Résumé :

Cette thèse construit un modèle théorique qui renouvelle l'approche traditionnelle de l'équilibre du marché. En introduisant dans le paradigme néo-classique le principe de préférence pour la quantité, il génère de façon optimale des stocks dans un marché concurrentiel. Les résultats sont très importants, car ils expliquent à la fois l'émergence des invendus et l'existence de cycles économiques. En outre, il étudie le comportement optimal du monopole dont la puissance de marché dépend non seulement de la quantité de biens étalés, mais aussi de celle de biens achetés. Contrairement à l'hypothèse traditionnelle selon laquelle le monopoleur choisit le prix ou la quantité qui maximise son profit, il attire, via un indice de Lerner généralisé la demande à la fois par le prix et la quantité de biens exposés. Quelle que soit la structure du marché, le phénomène d'accumulation des stocks de biens apparaît dans l'économie. De plus, il a l'avantage d'expliquer explicitement les achats impulsifs non encore traités par la théorie économique. Pour vérifier la robustesse des résultats du modèle théorique, ils sont testés sur des données américaines. En raison de leur non-linéarité, la méthode de Gauss-Newton est appropriée pour analyser l'impact de la préférence pour la quantité sur la production et l'accumulation de biens, et par conséquent sur les prévisions de PIB. Enfin, cette thèse construit un modèle à générations imbriquées à deux pays qui étend l'équilibre dynamique à un gamma-équilibre dynamique sans friction. Sur la base de la contrainte de détention préalable d'encaisse, il ressort les conditions de sur-accumulation du capital et les conséquences de la mobilité du capital sur le bien-être dans un contexte d'accumulation du stock d'invendus.

Mots clés :

Comportement Microéconomique, Comportement des entreprises, la théorie économique du Consommateur, Concurrence Parfaite, Monopole, Fluctuations du PIB, Régression non Linéaire, Méthode de Gauss-Newton, Stock d'Invendus, Détention préalable d'Encaisse, Modèle à Générations Imbriquées, Mobilité du Capital, Politique Monétaire.

Abstract :

This thesis constructs a theoretical model that renews the traditional approach of the market equilibrium. By introducing into the neoclassical paradigm the principle of preference for quantity, it optimally generates inventories within a competitive market. The results are very important since they explain both the emergence of unsold goods and the existence of economic cycles. In addition, it studies the optimal behavior of a monopolist whose the market power depends not only on the quantity of displayed goods but also that of goods that the main consumer is willing to buy. Contrary to the traditional assumption that the monopolist chooses price or quantity that maximizes its profit, through a generalized Lerner index (GLI) it attracts customers' demand by both the price and the quantity of displayed goods. Whatever the market structure, the phenomenon of inventory accumulation appears in the economy. Furthermore, it has the advantage of explicitly explaining impulse purchases untreated by economics. To check the robustness of the results, the theoretical model is fitted to U.S. data. Due to its nonlinearity, the Gauss-Newton method is appropriate to highlight the impact of consumers' preference for quantity on production and accumulation of goods and consequently GDP forecast. Finally, this thesis builds a two-country overlapping generations (OLG) model which extends the dynamic OLG equilibrium to a frictionless dynamic OLG gamma-equilibrium. Based on the cash-in-advance constraint, it highlights the conditions of over-accumulation of capital and welfare implications of capital mobility in a context of accumulation of stock of unsold goods.

Keywords :

Microeconomic Behavior, Firm Behavior, Economic Theory of the Consumer, Perfect Competition, Monopoly, GDP Fluctuations, Nonlinear Regression, Gauss-Newton Method, Stock of Goods, Cash-in-Advance, Overlapping Generations Model, Capital Mobility, Monetary policy.

Main Abbreviations

CES: Constant Elasticity of Substitution

CIES: Constant Intertemporal Elasticity of Substitution

CIPI: Change in Private Inventories or Change in Inventory Investment

GDP: Gross Domestic Product

GNP: Gross National Product

GNR: Gauss-Newton Regression

IMRS: Intertemporal Marginal Rate of Substitution

IT: Information Technology

KKT: Karush-Kuhn-Tucker

RBC: Real Business Cycles

NLLS: Nonlinear Least Squares

ML: Maximum Likelihood

OLG: Overlapping Generations

OLS: Ordinary Linear Least Squares

Summary

Introduction	19
0.1 Introduction	21
I Stock of Unsold Goods in Competitive and Monopoly Markets	33
0.2 Introduction	35
1 Competition and Stock of Unsold Goods : A Demand-based Approach	39
1.1 Introduction	40
1.1.1 Related literature	40
1.1.2 Motivation and objective of the chapter	43
1.1.3 Methodology	46
1.1.4 Results	47
1.2 The Model	48
1.2.1 Consumer Behavior : the demand side	49
1.2.1.1 The main consumer’s behavior	50
1.2.1.2 The corner solution : the rational buyer	52
1.2.1.3 The interior solution : the impulse buyer	57
1.2.1.4 The residual consumer	61
1.2.2 Competition	61
1.2.2.1 The competitive producer : the supply side	62
1.2.2.2 Corner solution	63
1.2.2.3 Interior solution	65

1.2.3	The Competitive γ -equilibrium	69
1.2.4	The functioning of the competitive selling-off market	74
1.3	Alternative Modeling	76
1.3.1	The case of two goods	77
1.4	Applications with usual utility functions	80
1.4.1	Preferences	80
1.4.2	The consumer's demand for goods and for displayed goods	82
1.4.3	The producer's price	83
1.4.3.1	The competitive price	84
1.4.4	The selling-off market	85
1.4.4.1	Curiosities	85
1.5	Conclusion	87
2	Monopoly and Stock of Unsold Goods : A Based-demand Approach	89
2.1	Introduction	90
2.1.1	Related literature	90
2.1.2	Motivation and objective of the paper	91
2.1.3	Methodology	93
2.1.4	Results	93
2.2	The Model	94
2.2.1	The Consumer Behavior	95
2.2.2	Monopoly	98
2.2.2.1	The monopoly producer	99
2.2.2.2	The monopoly γ -equilibrium	105
2.2.3	The external monopolist selling-off firm	107
2.3	Alternative modeling	108
2.3.0.1	The case of two goods	109
2.4	Applications with usual utility functions	112
2.4.1	Preferences	113
2.4.2	The consumer's demand for goods and for displayed goods	113

2.4.3	The producer's price	115
2.4.3.1	The monopoly price	115
2.4.4	The optimal stocks	119
2.4.5	The selling-off market	122
2.4.5.1	Curiosities	122
2.4.5.2	Economic interpretations	124
2.5	Discussion	124
2.5.1	International price non-convergence and international migration	125
2.5.2	Involuntary unemployment and the Beveridge curve	125
2.5.3	Monetary policy and the Phillips curve	126
2.6	Conclusion	129
.1	Conclusion	132

II Econometric Evidence and Overlapping Generations Economy 135

.2	Introduction	137
A	Preference for quantity : A Based-demand Empirical Approach 141	
A.1	Introduction	142
A.2	Theoretical Model	144
A.2.1	Households	145
A.2.1.1	Corner solution	146
A.2.1.2	Impulse buying	148
A.2.2	Firms	148
A.2.3	The competitive price	148
A.2.4	The monopoly price	149
A.3	Econometric Considerations	150
A.3.1	Nonlinear Regression	150
A.3.1.1	The Gauss-Newton Method	154
A.4	Data and Empirical Results	155

A.4.1	Restricted Model	165
A.5	Conclusion	167
A.6	Appendix	168
A.6.1	CES function	168
A.6.2	Testing Trinity	169
A.6.3	Estimation Results	170

B A Two-country Overlapping Generations Model with Stock of unsold goods and Liquidity Constraint 177

B.1	INTRODUCTION	178
B.2	THE MODEL	181
B.2.1	<i>Households</i>	181
B.2.1.1	Cash in Advance Constraint and Investment Abroad . . .	183
B.2.1.2	Consumption and Savings	185
B.2.1.3	The Demand for Money	189
B.2.2	<i>Production</i>	191
B.2.2.1	Capital Market γ -equilibrium	191
B.2.3	<i>National Accounting</i>	194
B.3	Autarky	196
B.3.1	Autarkic Inter-temporal γ -equilibrium	196
B.4	International Capital Mobility	199
B.4.1	The role of inflation on the stock of unsold goods at the γ -equilibrium	200
B.4.2	Steady-state Welfare Implications of capital with Stock of Unsold Goods	203
B.4.3	Steady-state welfare implications of capital mobility with stock of unsold goods and money	205
B.4.4	Exemple : CES-utility and Cobb-Douglas production functions . . .	206
B.4.5	Optimal Stationary Paths with Stock of unsold Goods	208
B.5	Monetary policy and Capital Mobility	210
B.5.1	Consumption and Savings	211

B.5.2	Capital Dynamics with Money Creation and Stock of Unsold Goods	213
B.5.2.1	Steady-state γ -equilibrium with Money Creation	214
B.6	Conclusion	214
B.7	Appendix	216
B.7.1	Examples of a CIES function	216
B.8	Conclusion	218
Conclusion		221
B.9	Aspect théorique	223
B.9.1	Existence de stock de biens dans une structure de marché concurrentiel	224
B.9.2	Existence de stock de biens dans une structure de marché monopolistique	225
B.10	Aspect empirique	226
B.10.1	Stock de Biens invendus : Cause des Fluctuations économiques . . .	227
B.11	Stock de biens dans une économie monétaire à Générations imbriquées . .	228
List of Figures		231
List of Tables		233
Index		235
Appendices		235
Bibliography		235

Introduction

"Economic knowledge is historically determined . . . what we know today about the economic system is not something we discovered this morning but is the sum of all our insights, discoveries and false starts in the past. Without Pigou there would be no Keynes; without Keynes no Friedman; without Friedman no Lucas; without Lucas no . . .", (Blaug, 1991a, pp. x-xi)

0.1 Introduction

"Simplified paradigms or maps are indispensable for human thought. On the one hand, we may explicitly formulate theories or models and consciously use them to guide behaviour. Alternatively, we may deny the need for such guides and assume that we will act only in terms of specific objective facts, dealing with each case on its own merits. If we assume this, however, we delude ourselves. For in the back of our minds are hidden assumptions, biases, and prejudices that determine how we perceive reality, what facts we look at, and how we judge their importance and merits", Huntington (1996).

L'équilibre est une notion fondamentale rencontrée dans la plupart des sciences. Ainsi, les astronomes utilisent les équilibres thermiques et hydrostatiques, les physiciens emploient l'équilibre statique et l'équilibre thermodynamique alors que les chimistes utilisent l'équilibre chimique¹. Il n'y a aucun doute que la théorie économique a depuis longtemps considéré également la notion d'équilibre comme un phénomène fondamental. De Smith (1776) à Walras (1870), la théorie de l'équilibre général approfondie par les travaux d'Arrow-Debreu (1954) et Debreu (1959) suppose que l'économie atteint un état pour lequel le prix du marché est considéré comme un prix d'équilibre s'il permet l'égalité entre l'offre et la demande de biens. Cependant, l'existence de ces prix d'équilibre dépend d'un certain nombre d'hypothèses selon lesquelles tous les individus (producteurs et consommateurs)

1. Astronomie : l'équilibre hydrostatique est l'un des plus importants principes fondamentaux de la physique de l'atmosphère et de l'astrophysique. Cet état d'équilibre existe lorsque la compression due à la gravité est équilibrée par une force de gradient de pression; *équilibre thermique* : l'énergie totale qui coule vers l'extérieur à un rayon donné (la luminosité à ce rayon) doit simplement correspondre au total l'énergie qui se produit à l'intérieur de cette couche. En Physique, l'équilibre est définie comme étant la condition d'un système dans laquelle ni son état de mouvement, ni son état intérieur de l'énergie ne tend à changer avec le temps. Un corps mécanique simple est dit en équilibre si elle connaît ni accélération linéaire ni accélération angulaire. En thermodynamique, il existe un équilibre thermique lorsque les réactions se produisent à des rythmes tels que la composition du mélange invariable par rapport au temps. Dans une réaction chimique, l'équilibre chimique est l'état dans lequel les deux réactifs et les produits sont présents à des concentrations qui n'ont pas tendance à changer avec le temps.

sont parfaitement informés, totalement rationnels (ils ne sont confrontés à aucune rationalité limitée, voire l'incertitude), tous les prix sont flexibles et il y a absence de coûts d'ajustement.

Ayant une dominance fondamentale au niveau de la théorie microéconomique néo-classique, la flexibilité des prix permet à long terme tant aux marchés concurrentiels que monopolistiques d'être en situation d'équilibre. Par exemple, tout déplacement vers le haut ou vers le bas de la courbe d'offre d'une entreprise évoluant dans un marché concurrentiel indique qu'elle peut produire plus ou moins en réponse à une augmentation ou une diminution du prix du marché respectivement. En d'autres termes, cette situation peut se résumer par le fait que la réponse de la firme maximisant son profit à tout mouvement au niveau des prix relatifs est considérée comme une réaction normale. Comme il y a rationalité et information parfaite, le comportement de la firme est nécessairement compatible avec le comportement des consommateurs qui, de leur côté, cherchent à maximiser leur fonction d'utilité par rapport à un ensemble de consommation disponible. Cette interaction entre les consommateurs et les producteurs détermine les conditions des marchés dans un contexte caractérisé notamment par une structure purement concurrentielle et/ou monopolistique.

Dans un système de marché, les prix de tous les biens — y compris ceux de l'argent, du capital et du travail — sont interdépendants. Un changement dans le prix d'un bien sur un marché quelconque peut avoir un impact sur un autre. D'après la loi de Walras, si tous les marchés sauf un sont en équilibre, alors le dernier marché doit être également en équilibre. En d'autres termes, si les $n - 1$ marchés sont en équilibre, alors le n -ième, l'est aussi. Se référant à une proposition plus faible, cela signifie que la valeur totale de la demande excédentaire ne peut dépasser celle de l'offre excédentaire, Florenzano (1987). Dans le cas contraire, le marché sera dit en situation de déséquilibre. Cependant, le sens de ce dernier ne doit pas s'interpréter comme un état dans lequel les forces du marché agissent de manière à changer les valeurs actuelles des variables endogènes. En effet, cette situation de déséquilibre de marché peut représenter la solution d'un modèle plus général dans lequel les agents agissent de manière optimale, compte tenu de l'ensemble des contraintes auxquelles ils sont confrontés, Quandt (1988). C'est le sens de l'équilibre sur lequel nous

mettons l'accès dans cette thèse.

Cet état d'équilibre s'applique à plusieurs structures de marchés dont la concurrence pure et parfaite et le monopole pur. La concurrence pure et parfaite qui est considérée comme la situation idéale par rapport à laquelle se mesurent le monde réel et la concurrence imparfaite, est le centre du paradigme néoclassique utilisant le concept de base de l'équilibre du marché où le prix fixé par une entreprise est déterminé de telle sorte que la demande est égale à l'offre du marché. Dans un tel paradigme concurrentiel, tous les biens sont évalués à leur prix d'équilibre. Donc, nous ne pouvons pas prendre en compte l'existence des stocks de biens invendus, notamment les stocks invendables ou marchandises invendables. À court terme, les entreprises en concurrence pure et parfaite maximisent leur profit, alors que les consommateurs maximisent leur fonction d'utilité sous leur contrainte budgétaire. Dans ce cas, le comportement du consommateur peut influencer l'équilibre du marché. Cependant, sur le long terme la demande et l'offre de biens affectent l'équilibre concurrentiel et toutes les entreprises considérant les prix comme donnés feront un profit normal, ce qui veut dire qu'ils feront un profit nul.

Contrairement à la structure de marché purement et parfaitement concurrentielle, celle du marché monopolistique suppose que l'entreprise a un droit de vente exclusif. Ainsi, le pouvoir du monopoleur peut avoir un impact sur ses produits compte tenu de l'influence de son plan de production sur le prix de sa production. La quantité de biens produits par le monopoleur relativement à son pouvoir de marché répond de manière continue comme une fonction du prix qu'il fixe. Sous l'hypothèse de l'information parfaite et de la flexibilité des prix, elle fait face à des contraintes liées au comportement des consommateurs. Comme c'est le cas du marché concurrentiel, les caractéristiques du monopole sont très importantes pour l'analyse de la structure de marché puisqu'elles nous permettent d'étudier la structure normative des autres structures de marchés.

Bien que l'accent soit mis dans la thèse sur les deux structures extrêmes de marché, la concurrence pure et parfaite et le monopole pur, nous savons qu'il y a dans la littérature économique d'autres structures de marché permettant d'analyser les équilibres statique et dynamique, à savoir, la concurrence monopolistique, l'oligopole, le duopole, oligopsonne et le

monopsonne. Cependant, nous n'étudions que la concurrence pure et parfaite et le monopole afin d'analyser théoriquement les conditions d'interaction entre les consommateurs et les producteurs et celles d'équilibre. A cet effet, nous convainquons que si les résultats sont valables pour les deux structures des marchés concurrentiels et monopolistiques, alors ils seront probablement vrais dans ces structures des marchés intermédiaires qui sont plus proches de la réalité.

Quelle que soit la structure du marché, concurrentielle ou monopolistique, la notion d'équilibre est traitée du point de vue à la fois statique et dynamique. Un équilibre statique est défini comme étant l'état dans lequel toutes les variables ou les quantités d'une économie restent inchangées par rapport au temps. A cet effet, le temps n'est pas pertinent à l'explication des phénomènes économiques. Toutefois, la question du temps est tout à fait pertinente dans le cas des études de l'équilibre général dynamique qui est vu comme une situation dans laquelle toutes les indicateurs économiques varient au même rythme au cours du temps. Dans ce système dynamique, avec la prise en compte de l'information parfaite, la certitude et la flexibilité des prix, les marchés des biens, financier et monétaire sont en équilibre. Ceci est particulièrement possible dans le cas des deux principaux modèles de micro-fondée dans la littérature économique.

Le premier modèle dynamique, basé sur celui de Ramsey (1928), analyse le modèle de croissance économique optimale en situation de certitude, en dérivant les conditions inter-temporels qui sont satisfaits sur la trajectoire de croissance optimale choisie par un planificateur social, tandis que le second portant sur le modèle à générations imbriquées d'Allais (1947), Samuelson (1958) et Diamond (1965) étudie l'économie dans son ensemble du point de vue dynamique. Ces modèles sont utilisés dans des approches basées sur la micro-fondation de la macroéconomie. Dans ces types de modèles, l'équilibre concurrentiel dynamique permet la détermination du niveau des prix pour lesquels la production et du stock de capital croissent au même rythme. Du point de vue statique ou dynamique, dans ce paradigme économique, tous les marchés sont aussi en situation d'équilibre. Cependant, les stocks de biens invendus existent dans l'économie du monde réel.

Il s'agit d'un problème économique réel à prendre en considération puisque les données

observées tant aux Etats-Unis ainsi que dans la Zone européenne permettent de se pencher sur l'existence de stock de produits invendus, qui, dans la plupart des cas se terminent en stock de biens invendables ou non échangeables. Les exemples sont nombreux sur presque tous les marchés, même si la liste de toutes les entreprises faisant face à type de problème n'est pas mentionnée ici.

En effet, aux Etats-Unis, les stocks de biens invendables existent surtout dans les supermarchés. Par exemple, en Octobre 2011, la valeur des biens invendus non durables ont augmenté de 2,8 % alors que l'achat a augmenté de 1,7 %. En Europe, les stocks de biens invendables dans l'industrie des vêtements sont évalués à environ 20 % de la production totale par an, dans l'industrie des journaux 36 % par an, CD / DVD 40 % par an.

En 2010, le chiffre d'affaires global de Michelin était de 17.89 milliards d'euros. La valeur du stock de biens invendus était 3.77 milliards d'euros, soit 21.07 % de son chiffre d'affaires global. Voici des années précédentes (Source : Chiffres Clés du Guide MICHELIN 2010) :

Année	Recette totale en milliard d'euros	Stocks de biens invendus en milliard d'euros	stock / Recette totale
2006	16.38	3.34	20.39%
2007	16.87	3.35	19.86%
2008	16.41	3.68	22.42%
2009	14.81	2.99	20.19%
2010	17.89	3.77	21.07%

L'entreprise avait prévu de terminer l'année 2010 avec un taux 16 % des stocks d'invendus, mais un tel chiffre n'a pas été réalisé. Les gestionnaires savent que la demande reste insatisfaite, mais n'ont pas de solutions opérationnelles pour pouvoir dégager le marché de ces invendus qui touchent presque tous les secteurs. Les consommateurs achètent de plus en plus pendant la période des soldes, au cours de laquelle les entreprises vendent leurs produits à des prix de marge réduite. L'exemple de la compagnie MICHELIN illustre bien l'existence d'un nouveau phénomène empirique communément observés sur les différents marchés à travers le monde.

L'existence de ces stocks de marchandises invendus constitue une perte pour l'économie en termes de valeur ajoutée. En effet, les invendus correspondent à une recette totale non réalisée par une entreprise quelconque. Plus le stock d'invendus est important moins le chiffre d'affaire est important, ce qui engendre la baisse des profits des entreprises. Cette situation diminue sous doute la capacité d'embauche de l'entreprise. Comme résultat, elle peut générer, sur le plan macroéconomique, plus de chômage, et par conséquent plonger l'économie dans la récession.

Étant une source de fluctuations économiques, les stocks d'invendus généralement inclus dans la catégorie des investissements en stocks ou les stocks de biens utilisés par les entreprises, représentent une composante non négligeable de volatilité du Produit Intérieur Brut (PIB) en termes réels. Par ailleurs, le co-mouvement entre le stock de biens et les ventes finales ainsi que la variance élevée de la production par rapport à celle des ventes permettent de mettre en exergue le rôle de ces stocks de biens invendus sur les cycles réels (RBC) et les fluctuations économiques. Ceci est alors compatible avec La théorie des cycles réels qui considère les périodes de récessions et de croissance économique comme une réponse efficace à des changements exogènes dans l'environnement économique réel.

Une meilleure compréhension des facteurs déterminant l'émergence des invendus dans l'économie est très importante car elle aide le gouvernement à mettre en œuvre des politiques économiques basées sur une approche micro-fondée. La question est de savoir comment prendre en compte ces stocks de biens dans le cadre du paradigme néoclassique ? À cet égard, nous introduisons le principe de préférence pour la quantité dans le paradigme néoclassique caractérisé par l'information parfaite, la rationalité parfaite, l'absence d'incertitude et la flexibilité des prix. Ce principe de préférence pour la quantité permet de capturer l'appréciation du consommateur en termes d'utilité de la quantité disponible de bien affichée qu'il décide de ne pas acheter².

2. Notez que nous utilisons la notion de principe, et non celle de l'hypothèse. En effet, une hypothèse se réfère à l'acte de prendre quelque chose pour acquis ou quelque chose qui va de soi, alors que le principe est une vérité générale et fondamentale qui peut être utilisée pour décider de la conduite ou du choix, c'est une règle d'action.

Par exemple, le principe d'Archimède n'est pas une hypothèse au même titre que le principe de la minimisation de l'énergie n'est pas une hypothèse en physique, la maximisation de l'utilité des consom-

L'introduction de la préférence pour la quantité dans le paradigme traditionnel génère deux types de demande différents du côté des consommateurs. Une demande qui permet de capter de la quantité que le consommateur prévoit d'acheter (comme d'habitude), une autre qui correspond à la quantité de bien affichée que le consommateur principal souhaite voir lors de l'achat. Le deuxième type de demande n'a jamais été étudié par la théorie économique, malgré que son existence a été empiriquement montré par de nombreuses études en marketing. La première catégorie de demande définit les ventes, tandis que la seconde impose à l'entreprise une contrainte sur la quantité qu'elle doit produire pour attirer la demande. La présence des stocks de biens invendus pour lesquels il n'y a plus de demande, soulève la question de savoir pourquoi les prix ne baissent pas afin de diriger l'économie sur son état d'équilibre.

Cette thèse développe un modèle théorique permettant d'introduire dans une économie néoclassique le principe de préférence pour la quantité basé sur le concept de biens étalés utilisés couramment en management. Il nous permet d'étendre l'équilibre néoclassique traditionnel, où les stock d'inventures sont théoriquement absents à un nouvel équilibre, appelé γ -équilibre où les stocks d'inventures sont théoriquement présents. Ce nouveau concept de γ -équilibre comporte l'équilibre concurrentiel néoclassique comme un sous-cas. Il explique l'émergence et la persistance des stocks d'inventures dans les structures de marché à la fois concurrentielle et monopolistique comme un phénomène d'équilibre sous les hypothèses de l'information parfaite, l'absence d'incertitude, la flexibilité des prix et la rationalité complète des entreprises et des consommateurs. Dans ce nouvel équilibre de marché, il est admis que le comportement du consommateur principal génère un stock des marchandises pour lesquelles il n'y a plus de demande.

Il est important de souligner qu'il existe plusieurs manières de modéliser le principe de la préférence pour la quantité. Nous avons opté dans ce travail de recherche pour la façon la plus simple et la plus tractable. En effet, du point de vue empirique, une telle préférence pourrait s'expliquer par le niveau du salaire et de la richesse d'un individu représentatif, son

moteurs ou du profit des entreprises n'est pas une hypothèse en économie, mais plutôt un principe en soi.

âge, son sexe, sa zone urbaine ou rurale, ses habitudes, les normes sociales, les conditions sociales dans lesquelles il vit, etc. En outre, dans le cas des produits hétérogènes des effets croisés entre les produits peuvent également être pris en compte à travers les élasticités croisées, comme c'est déjà fait en gestion. La préférence pour la quantité peut se voir différemment en fonction des différents types de biens qu'un individu choisit de consommer.

Cette thèse est divisée en deux grandes parties. La première partie intitulée *Stock de biens invendus dans les Marchés Concurrentiel et Monopolistique* contient deux chapitres dont l'objectif principal est de construire un modèle théorique montrant l'existence d'invendus dans un paradigme néoclassique. Ces deux premiers chapitres sont issues de travaux recherche réalisés en collaboration avec le professeur Joël Blot (Université Panthéon-Sorbonne, Paris I) et professeur Damien Gaumont, mon directeur de thèse de doctorat (Université de Panthéon-Assas, Paris II). Dans le premier chapitre, nous développons un modèle de base qui montre l'existence des stocks de biens invendus dans un marché concurrentiel tandis que le second traite le cas du marché monopolistique. Dans le cas du monopole, nous construisons un nouvel indice, appelé indice de Lerner augmenté qui prend en compte le principe de préférence pour la quantité. Cet indice permet de mettre l'accent sur les variables stratégiques relatives au pouvoir du monopole.

La deuxième partie de la thèse, intitulée *Considérations Empiriques et Modèle à Générations Imbriquées*, a un double objectif. D'une part, elle vise à appliquer les résultats du modèle théorique développé dans la première partie aux données trimestrielles américaines. Ces considérations empiriques nous permettent d'expliquer l'influence des stocks de marchandises invendues, sur les fluctuations de l'économie américaine. D'autre part, cette partie construit un modèle à générations imbriquées à deux pays dans le but de déterminer les implications de la mobilité du capital pour le bien-social relativement au principe de préférence pour la quantité et à la politique monétaire. Les paragraphes suivants exposent les différentes de la thèse.

- Le premier chapitre vise à établir une approche concurrentielle micro-fondée avec information parfaite, la flexibilité des prix, sans aucune incertitude et capable de reproduire à long terme, les régularités suivantes : la production totale dépasse les

ventes, il y a co-mouvement entre la production totale et les ventes finales, la volatilité de production est plus importante que celle des ventes finales, les entreprises cherchent à liquider leurs stocks d'invendus sur les marchés du déstockage mais elles sont confrontées à des stocks de biens invendables qui finissent par se transformer en dons (à des organisations caritatives), se détruire ou se recycler. Pour atteindre cet objectif, notre modèle met l'emphase sur la quantité de biens exposés, avec l'idée que ce dernier permet de stimuler directement la demande. Il est important de souligner que cette démarche repose la littérature liée au marketing.

- Le deuxième chapitre vise à étudier le comportement optimal d'un monopoleur dont le pouvoir de marché dépend non seulement de la quantité de biens imputables au principe de préférence pour la quantité, mais de la quantité de biens que le consommateur principal est prêt à acheter pendant la période de marché. Contrairement à l'hypothèse traditionnelle selon laquelle le monopoleur choisit de façon optimale le prix ou la quantité qui maximise son profit, ce chapitre présente un cas intéressant où il résout son problème de maximisation par rapport au comportement des consommateurs, via notamment le principe de la préférence pour la quantité. Par conséquent, le stock d'invendus disponibles à la fin de la période de marché est influencé à la fois par le comportements des consommateurs et celui de l'entreprise monopolistique. En analysant le rôle des stocks d'invendus sur le marché, nous définissons un indice modifié de Lerner expliquant le degré de pouvoir de marché du monopoleur. S'il n'y a pas de stock de produits invendus, alors l'indice de Lerner est équivalent à celui vu dans la littérature, mais en présence d'invendus, il illustre parfaitement comment les entreprises manipulent les prix ou les biens affichés pour attirer la demande des consommateurs.
- Le chapitre trois s'inscrit dans le cadre d'une démarche basée essentiellement sur des considérations empiriques. Il s'agit de montrer comment le principe de préférence pour la quantité influence de façon significative la production (ou le PIB) ainsi que le stock de biens. En utilisant les données trimestrielles américaines allant du premier trimestre 1995 au troisième trimestre 2011, il permet de mettre en exergue le

rôle important des stocks d'inventures dans l'évolution des cycles réels (RBC) à travers l'estimation des paramètres pertinents à l'émergence de ces stocks d'inventures dans l'économie. A cet effet, il est admis dans la version théorique développée dans la première partie de la thèse que ce nouveau concept économique relatif à la préférence des consommateurs joue un rôle fondamental dans l'explication des comportements optimaux des consommateurs et des entreprises évoluant dans une structure de marché tant concurrentielle que monopolistique. Il en résulte que ce paramètre pertinent est mesurable quelle que soit la structure des marchés sous les hypothèses que l'environnement économique est caractérisé, entre autres, par l'information parfaite, la flexibilité des prix et l'absence de coûts d'ajustement. Pour mettre en œuvre ces mécanismes, il va falloir faire appel à une modélisation économétrique plutôt non linéaire (non seulement par rapport aux paramètres du modèle mais également par aux variables) basée sur l'algorithme de Gauss-Newton (Marquadt, 1963 ; Gallant, 1975).

- Le quatrième et dernier chapitre de la thèse vise à expliquer l'émergence des stocks d'inventures impulsée notamment par le principe de préférence pour la quantité via un modèle à générations imbriquées (OLG). Basé sur l'hypothèse de détention préalable d'encaisse (cash-in-advance), ce modèle OLG à deux pays analyse la manière dont les ménages choisissent de laisser leur consommation par rapport au taux marginal de substitution intertemporal pondéré sur le paramètre indiquant le principe de préférence pour la quantité. Dans ce chapitre, nous partons de l'hypothèse que ce dernier est plus prononcé dans le pays qui exporte du capital que celui qui en importe (pays étranger). De plus, nous analysons les conditions de stabilité locale dans le cas d'une économie en autarcie avec l'existence des stocks de biens et du stock de monnaie. Dans le contexte de mobilité internationale du capital, où l'hypothèse de l'autarcie est relâchée, nous analysons la trajectoire optimale de l'équilibre stationnaire en présence des stocks de biens et la détention préalable d'encaisse. Aussi, détermine-t-on, d'une part, les conditions de l'existence de cette mobilité, et d'autre part, celles de la règle d'or (*Golden rule*), et par conséquent de l'accumulation du capital lorsque le principe de préférence pour la quantité ne joue qu'à la première de vie des indivi-

dus. Enfin, ce chapitre vise à étudier l'impact de la politique monétaire tant sur la dynamique du capital que sur celui du bien-être des individus nés à la période t .

Première partie

Stock of Unsold Goods in Competitive and Monopoly Markets

0.2 Introduction

Observed data suggest that a lot of firms face the stock of unsold goods at the end of their market period. This stock of unsold goods – which is not a strategic stock for business – raises the question of why prices do not fall to clear goods markets as in neoclassical theory. An understanding of factors determining the emergence of the stock of unsold goods in the economy is very important since it helps government to implement economic policies based on a micro-founded approach. The question is how to account for the stock of unsold goods in neoclassical paradigm?

We introduce into economics the principle of preference for quantity based on the management science concept of facings. This demand-based approach allows us to extend the traditional neoclassical equilibrium to a γ -equilibrium which is considered as an equilibrium compatible with the stock of unsold goods and includes the traditional neoclassical equilibrium as a sub-case. It explains the emergence and persistence of stock of unsold goods as an equilibrium phenomenon with perfect information, no uncertainty, price flexibility and full rationality of both firms and consumers. Based on these assumptions, consumers's behavior depends on a principle of preference for quantity. This principle allows us to capture the consumer's valuation in terms of utility of the available quantity of the displayed good he decides not to buy.

The present part of the thesis is divided into two theoretical chapters. In the first chapter, we develop a basic model which shows that the stock of unsold goods exists in the competitive market structure while the second one is based on a monopolistic market structure.

The chapter one aims at building a competitive micro-founded approach with perfect information, price flexibility, without uncertainty, capable of reproducing in the long run the regularities that total production exceeds sales, total production co-moves with final sales, production volatility is greater than sales volatility, firms try to clear their stock of unsold goods on selling-off markets, failing which they face dead stocks, which end up being donated (to charities), destroyed or recycled. In this regard, our model focuses on

displayed goods, with the idea that displayed goods directly stimulates demand, according to the marketing literature.

The theory developed in this document has several important implications. On the one hand, our modeling generates the traditional competitive equilibrium without stocks of unsold goods for which any selling-off activities are not necessary and the competitive γ -equilibrium with stocks of unsold goods that may not clear on a well-organized selling-off market. In this equilibrium, displayed goods become a firm's strategic variable. On the other hand, our modeling has the advantage of describing, explaining, and forecasting a lot of observable phenomena. This is important because due to the existence of stock unsold goods which equivalently ends up with a long-term unemployment on labor market, quite a lot of value-added is not realized and consequently, not reinvested in the economy for production or inventories.

Another market structure that allows us to analyze the equilibrium, namely static equilibrium is the monopoly. In this market structure, the firm has the right of exclusive sale. The monopolist power may impact her goods, by taking into account the influence of her production plan on her output price. As it is the case of the competitive market, the monopoly features are very crucial while studying the market structures and allow us to normatively analyze the other extreme market structure.

The primary objective of the second chapter of this part is to investigate the optimal behavior of a monopolist whose the market power depends not only on the quantity of goods related to the principle of preference for quantity defined in chapter one, but on the quantity of goods that the main-consumer is willing to buy on the market period. Contrary to the traditional assumption that the monopolist optimally chooses the price or quantity that maximizes her profit, this chapter exhibits a case where the monopolist solves her maximizing problem with respect to the consumer behavior, particularly through the principle of preference for quantity. The stock of unsold goods available at the end of the market period is impacted by both consumer behavior and monopolist behavior.

The results have many implications. First, our modeling generates the traditional monopoly equilibrium without stock of unsold goods and the monopoly γ -equilibrium with

stock of unsold goods that may not clear on the well-organized selling-off market. Second, while analyzing the role of unsold goods on monopoly market we define a modified Lerner index of the firm's power. The Lerner index compatible with the stock of unsold goods perfectly illustrates how firms manipulate prices or displayed goods to attract consumers' demand. When consumers are budget-constrained, price is the relevant strategic variable, but when they are not, the quantity of displayed good becomes the strategic variable. Combining both types of consumers leads to a two-dimensional strategic behavior of firms manipulating both price and quantity of displayed good to attract both rational buyers and impulse buyers.

It is important to underline that there are various ways to model the preference for quantity principle. We choose the simplest one which has the property to be very simple and tractable. Indeed, from empirical observations, such a preference is expected to be a function of both the individual's wage and wealth, his age, sex, urban or rural style of live, country habits, social norms, social conditions and so on. Moreover, in the case of heterogeneous goods cross-effects between goods may also be taken into account by cross elasticities, as already made in management sciences. The preference for quantity is differently active depending on the various types of goods an individual chooses to consume.

1 Competition and Stock of Unsold Goods : A Demand-based Approach

Abstract

Based on the management science concept of facings, we introduce into economics the preference for quantity principle, which allows us to extend the neoclassical equilibrium to a frictionless γ -equilibrium. Such an equilibrium is compatible with stocks of unsold goods and contains as a sub-case the competitive equilibrium. It reproduces in the long run the following observed GDP regularities : total production exceeds sales, total production co-moves with final sales and production may be more volatile than sales.

JEL Classification : D11, D21, D41, D42.

Key words : Microeconomic behavior, firm behavior, economic theory of the consumer, perfect competition.

"When he came to Fifth Avenue, he kept his eyes on the windows of the stores he passed. There was nothing he needed or wished to buy ; but he liked to see the display of goods, any goods, objects made by men, to be used by men. He enjoyed the sight of a prosperous street ; not more than every fourth one of the stores was out of business, its windows dark and empty." Atlas Shrugged by Ayn Rand, Random House, October 10, 1957, USA.

1.1 Introduction

The presence of dead stock, for which there is no longer any demand, raises the question of why prices do not fall to clear goods markets. In this theoretical paper, we introduce into economics the preference for quantity principle based on the management science concept of facings. It allows us to extend the traditional neoclassical equilibrium (in which stocks are theoretically absent) to a γ -equilibrium (in which stocks are theoretically present). This γ -equilibrium contains the traditional competitive neoclassical equilibrium as a sub-case. It explains the emergence and persistence of dead stock as an equilibrium phenomenon with perfect information, no uncertainty, price flexibility and full rationality of both firms and consumers.

This introduction is organized as follows : Subsection 1.1 is devoted to the related literature on dead stock. It surveys the neoclassical approach without dead stock, and its extensions in RBC which points out the role of dead stock in the economic cycle. Finally, it reviews the management and marketing literature in which dead stock exists and has long been studied, through the concepts of shelf-store and facing of products. Subsection 1.2 deals with the motivation and the objective of the paper. Subsection 1.3 is devoted to definitions and methodology. Subsection 1.4 gives both results and paper organization.

1.1.1 Related literature

Under the traditional competitive paradigm, in which all goods are priced at their equilibrium value, there is no dead stock. Indeed, in this paradigm with perfect information, pure rationality and price flexibility, the competitive firms optimize their profit in the short run, whereas consumers maximize their utility function under budget constraints. The consumer's demand is generally price and income dependent. Whatever the market

structure is, in static equilibrium, the price that clears the market is such that supply equals demand (Smith (1776), Walras (1874), Arrow-Debreu (1954), Uzawa (1961)). Extensions to temporary equilibrium (Grandmont (1975)) and to the dynamics of general competitive equilibrium have been thoroughly considered, via the fiction of the Walrasian auctioneer (Arrow and Hurwicz (1958)). At time 0, the Walrasian auctioneer calls out some vector combining both spot and future prices. Buyers make price offers to him. He transmits this information to all the economic agents involved. Sellers react to these offers by making counter-offers. The price moves in the direction of excess demand, which is encapsulated in the so-called Tâtonnement process. When the clearing market price vector is found, exchange of goods takes place, so that no stock is left without being matched to a buyer. However, overlapping generation models study the dynamics of the whole economy (Allais (1947), Samuelson (1958), Diamond (1965), and for a more recent account of this literature, see De La Croix and Michel (2002)). Within this dominant neoclassical paradigm, with perfect information there is no means to account for dead stock in equilibrium. But dead stock does exist, so that every effort should be made to preserve the centrality of the neoclassical approach, even if that means rethinking the concept in the light of recent challenges.

Departing from the competitive paradigm the economic literature proposes various ways to account for unsold goods in economics. The first one is to consider a certain demand, see Shaffer (1991), Mathewson and Winter (1987) within the resale price maintenance context, or Kawasaki, McMillan, and Zimmermann (1983), Carlton (1986) concerning sticky prices, or Lazear (1986) for clearance sale. The second one is to consider uncertain demand. In that line of research, the different explanations of inventory accumulation share some key assumptions, like the existence of a delay between production and sales (resulting in an uncertain demand, through the concepts of niche competitive game (Prescott (1975), Bryant (1980), Lucas and Woodford (1993), Eden (1990) and Dana (1993)), resale price maintenance (RPM) (Rey and Tirole (1986), Denecker *et al.* (1996)). These works are capable of explaining many empirical observations, including one that is important to economists : GDP fluctuations, Khan and Thomas (2007). These two authors develop an equilibrium business cycle model where non-convex delivery costs lead firms to follow (S, s) inventory

policies, in line with Herbert E. Scarf (1960). Their model reproduces two-thirds of the (pro)-cyclical variability of inventory investment as well as a countercyclical inventory-to-sales ratio and greater volatility in production than sales, see also Schutte (1984) with instantaneous production. Khan and Thomas (2007) also underline an important observation for our theory : the existence of co-movement between inventory investment and final sales.

Otherwise, in both management science and marketing, several theories account for the accumulation of inventories, especially the shelf-store literature that concentrate on the role of displayed good on sales. The first approach is due to Within (1957), who observes that displayed inventory can help induce greater sales. Based on a study of apparel retailers, Wolfe (1968) presents empirical evidence that sales of this kind of merchandise are roughly proportional to the displayed inventory. Levin *et al.* (1972) underline that "the presence of inventory has a motivational effect [...]. Large piles of goods displayed in a supermarket will lead the customers to buy more" (see also Silver and Peterson (1985)). Larson and DeMarais (1990) defined the term "psychic stock" to refer to "retail display inventory carried to stimulate demand". Battberg and Neslin (1990) consider the promotional effect of shelf stocks.

Parallel to this, the marketing literature on shelf space tends to view the relation between inventory and sales in terms of tactical decisions regarding the promotional mix and how to allocate scarce resources such as budget or shelf space (Corstjens and Doyle (1981)). After this paper, it has been generally accepted that shelf space allocation is a central problem in retailing. The authors observe : "Yet this problem has not been solved in a way which is both conceptually sound and practically operational. Most retailers today still allocate shelf space on the basis of the subjective experience of merchandisers, while a few use commercial systems which are clearly non-optimal" (Corstjens and Doyle (1981)). Maximizing total store profit within a geometric programming framework allows the authors to optimize the space allocation across product categories (width of the assortment)¹. A dynamic extension of this original approach can be found in Corstjens and Doyle

1. The formulation of the demand for each product group is defined as a multiplicative power function

(1983). Balakrishnan, Pangburn and Stavroulaki (2004) underline that large quantities of a product (where inventory is highly visible as stacks) can increase demand. They also incorporate inventory-dependent demand into their model, but they do not explain why inventory increases demand rates. They simply assume that the phenomenon holds in certain circumstances. Assuming demand to be certain, as Gurnani and Drezner (2000), Baker and Urban (1988) calibrate their exponential demand function, $q^d = \alpha i^\beta$, with $\alpha = 0.5$ and $\beta = 0.4$, where i captures the facing of a product. This means that in order to sell 2 units of a good, the corresponding quantity of displayed good should be approximately 32 units. Any space allocation model proposing to optimize retailer's profits must incorporate both the demand and cost sides of the profit equation. The demand impact comprises the "main effect" of the positive elasticity of unit sales with respect to increased shelf space that will normally exist within a store. It also involves cross effects both from the change affecting the relative display exposure of that item *vis-à-vis* all other products and from relations of substitution or complementarity between items.

1.1.2 Motivation and objective of the chapter

In addition to the literature that proposes a supply-side explanation of the existence of unsold goods through the assumption of uncertain demand (delay between production and sales), this paper provides an alternative demand-side deterministic explanation of the emergence of such stocks. The motivation for a deterministic explanation of the emergence of unsold goods is the following. First, it has the advantage of being simple, plausible and easily tractable in economics. Second, when it comes to explaining important phenomena, such as GDP fluctuations, we need to seek deeper explanations of the true causes of empirical observations, through deterministic modeling based on the economic behavior of individuals. We are not arguing that in reality firms do not face random demand, but that

of the display areas allocated to all of the product groups. The general form of their demand function Q^d is as follows : $Q_i^d = \alpha_i s_i^{\beta_i} \prod_{j=1, j \neq i}^K s_j^{\delta_{ij}}$. where α is a scaling parameter and β is the direct elasticity with respect to a unit of shelf space s_j . This parameter β has been measured using data on five product groups and is such that $\beta \in [-0.01, 0.19]$. Therefore cross-elasticities between products i and j explicitly enter into the picture through $\delta_{ij} \geq 0$, and K is the number of products. It is interesting to note that price does not enter the above demand function. This encompasses this particular case.

if we want to discover what is behind this uncertainty, we must momentarily abandon such an assumption. Randomness appears to be simply the result of the superposition of various deterministic phenomena. Our contribution possibly underlines one of these. More precisely, abstracting from the above mentioned marketing literature in which displayed inventories generates demand, we explain the reason why facing plays such a huge role in sales. For doing this, we introduce into economics the principle of preference for quantity².

Principle 1. The preference for quantity captures the consumer's valuation in terms of utility of the available quantity of the displayed good he decides not to buy.

Introducing the preference for quantity into the paradigm generates two separate types of consumer demand. One captures the quantity planned to buy (as usual), the other one captures the quantity of displayed good the main consumer needs to see while buying. The latter demand has never been theoretically studied, despite that its existence has been empirically exhibited by numerous studies in marketing. The first demand defines the sales, while the second imposes to the firm a production constraint. Indeed, firms have to take into account the demand for displayed goods (or facing), since it provides the consumer with a set of services that enters his utility function, which determines his demand for goods. Empirical evidence shows that displayed goods are complementary to consumption expenditure. The rationale behind this assumption is that stock of displayed goods reduces transaction and transport costs or matches consumers' demands more precisely. High inventories may stimulate the demand for various reasons. For example, tall stacks of a product can promote visibility, thus kindling latent demand. A large inventory might also signal a popular product, or provide consumers with an assurance of high service levels and future availability. Having many units of a product on hand also allows retailers to disperse the product across multiple locations on the sales floor, thereby potentially capturing additional demand (Balakrishnan, Pangburn, and Stavroulakis (2004)).

2. Note that we use the concept of Principle, and not the one of Assumption. Indeed, an assumption refers to the act of taking something for granted or something that is taken for granted, whereas a principle is a general and fundamental truth that may be used in deciding conduct or choice, it is a rule of action. For example, the Archimedean Principle is not an assumption exactly as the minimization of energy principle is not an assumption in physics, the maximization of utility or profit is not an assumption in economics but a principle in itself.

The idea that stocks of unsold goods enter the utility function is not new in economics. James A. Kahn, Margaret McConnell, and Gabriel Perez-Quiros (2002), studying GDP fluctuations in the USA during the period 1953-2000, consider the possibility that *inventories* are a source of household utility. The consumer side of their model provides the underlying motive for the target *inventory-to-sales* ratio. They claim that this would also be true for inventories at other stages of the production process, although they do not model them explicitly. Our theory is in line with the above modeling, except that in our case displayed goods (and not inventories) enter the utility function. This is important for our purpose with perfect information, since most of the time consumers really do not know the true level of inventories stored in the warehouse, but can, on the contrary, observe the quantity of displayed goods. If the displayed quantity of goods is high enough, consumers can freely decide how much to buy.³ If not, they may have to return at least once more to complete their shopping. Consumers buy a given quantity of good from the producer, who knows the consumer's preferences for quantity, so that he produces (stores/ displays) a larger quantity of goods. In doing so, he helps to stimulate demand (no one wants to enter an empty shop).

Due to this new extra demand, we are capable of defining an inventory-dependent demand rate which generates (or not) a stock of unsold goods (and, some times more dramatically, dead stock). The novelty here is that it comes from consumer preferences. We start with this principle because we wish to construct the simplest possible model focusing simply on the preference for quantity as a possible deterministic cause of persistence of stocks of unsold goods and dead stock. Section 3 proposes some extensions.

The objective of our paper is to build a micro-founded approach without uncertainty, capable of reproducing in the long run the following regularities : total production exceeds sales, total production co-moves with final sales, production volatility is greater than sales volatility, firms try to clear their stock of unsold goods on selling-off markets, failing which they face dead stocks, which end up being donated (to charities), destroyed or recycled. To

3. No one wants to buy the last shopworn apple.

our knowledge, no previous deterministic microeconomic modeling has explicitly succeeded in meeting all these points at once. In order to achieve this objective, our model focuses on displayed goods, with the idea that displayed goods directly stimulates demand, according to the marketing literature.

1.1.3 Methodology

The chapter keeps as relevant all the theoretical basic neoclassical assumptions (and in particular certainty). All the how it is possible to explain the existence of ex post dead stock using a demand side explanation. Neither uncertain demand, nor fixed price, nor transaction cost assumptions are necessary to generate equilibrium with stocks of unsold goods and dead stock. Moreover, even if we accept the assumption of random demand, we should observe that sales sometimes exceed production, generating a sustainable excess demand and no stock accumulation. But the known empirical regularities show that production (at the aggregate level) is always greater than sales in the long run. This reinforces the argument in favor of our quest for the possible deterministic causes of stock accumulation.

Formally, the extra demand for display goods (which appear in addition of the traditional demand for goods) emerges as follows. Let us define γ a parameter capturing the preference for quantity. Let us also define a function $q : [\underline{\gamma}, \bar{\gamma}] \rightarrow \mathbb{R}_+$ such that $\gamma \mapsto q(\gamma)$, with $q(0) = 0$. This function captures the impact of the preference for quantity on a market. It is important to note that $q(\gamma)$ is increasing with respect to γ . A small γ is associated with a small $q(\gamma)$, moreover $q(0) = 0$ implies that we are back to the traditional unique consumer's demand function for goods, which is a sub-case of our modeling.

Our framework holds for rational buys (the traditional corner solution, which is price-dependent) as well as for impulse buys (the interior solution which is price-independent). As will become clearer below, if the demand for goods and the demand for displayed goods intersect, they only intersect once at the traditional competitive equilibrium. For that reason, they play a huge role out of the traditional equilibrium. Consequently, our version of the the neoclassical paradigm encompasses the traditional competitive equilibrium as a

sub-case, and importantly has a descriptive and predictive content through the explanation of the pre-existing stock of unsold goods in the long run, absent from the traditional neo-classical paradigm. Otherwise, during the selling-off period, displayed goods are substitutes to consumption expenditure for the residual consumers. Those consumers have no particular preference for quantity, and behave traditionally. They simply enjoy consumption as the traditional neoclassical paradigm describes. Alternative models are also proposed in which the consumer may be a main consumer for some goods, and a residual consumer for other goods. Again, stocks of unsold goods appear, as well as dead stock.

1.1.4 Results

The theory we develop has several important implications. First, we show how market imperfection rationally emerges as a frictionless long run equilibrium. The model generates the traditional competitive equilibrium without stocks of unsold goods (and consequently, no need for any selling-off activities), and the competitive γ -equilibrium with stocks of unsold goods that may not clear on a well-organized selling-off market. In this equilibrium, displayed goods become a firm's strategic variable.

Second, our modeling has the advantage of describing, explaining, and forecasting a lot of observable phenomena. This is important because due to the existence of dead stock (or equivalently of long-term unemployment on labor market), quite a lot of value-added is not realized and consequently, not reinvested for production or inventories.⁴

The chapter is organized as follows. Section 2 presents the general model for competitive market structure. Section 3 is concerned with two alternative modelings, which are extensions of previous models. Section 4 presents different applications with usual utility functions (Log and *CES*), prior to Section 5 which concludes.

4. By *inventory* we mean any stock of saleable goods available at the beginning of the market period, for which a potential demand exists. This term encompasses finished goods and goods for resale. By *dead stock* we mean any stock of goods that remains unsold at the end of the selling-off period and for which no demand exists any more. Denoting by S/I the ratio of sales to inventories (the rate of inventory turnover), the dead stock ratio is determined by $1 - S/I$. If this ratio equals 0, then there is no dead stock.

If inventory coincides with production, the ratio S/I can stand for the rate of inventory investment. This is the difference between goods produced (production) and goods sold (sales) during a given period, and can be applied to the whole economy or to a firm.

1.2 The Model

The aim of this chapter is to change as little as possible the traditional behavior of both the consumer and the producer. We keep all the usual assumptions relative to the standard microeconomic behavior of the consumer and the producer. We do not introduce uncertainty, imperfect information or asymmetry of information, nor do we introduce price rigidity, adjustment costs or any type of complexity like wrong expectations or bounded rationality. Even under all these unfavorable assumptions, we show how stocks of unsold goods can emerge in the economy.

Subsection 1.2.1. deals with a firm that only produces and sells its products, but according to the preference for quantity principle drawn from marketing literature, the firm may be left with a stock of unsold goods. We first develop a simple model showing how a persistent stock of unsold goods exists. There is a single good, and consequently there is only one market, which is the competitive market. On this market structure, we analyze both the rational consumer (corner solution) and the impulse buyer (interior solution) and we show there are two possible equilibria. In the first, the model generates exactly the traditional neoclassical competitive equilibrium. In competition, firms enter the market until stocks clear. In the second type of neoclassical equilibrium — which does not co-exist with the first one — firms end the market period with a stock of unsold goods.

A key assumption here is that the main consumer has a preference for quantity. The main consumer buys a quantity of goods from the producer. Mindful of the main consumers' desire to see displays of goods (the facing in marketing) when shopping, the competitive firm produces a larger quantity of goods at the prevailing price, since this stimulates demand (no one wants to enter an empty shop). To capture the fact that the market period ends with a stock of unsold goods, we define the new concept of the market competitive equilibrium.

Subsection 1.2.2. deals with the external selling-off firm, which takes as given the previous stock of unsold goods and tries to sell it on the selling-off market. This is a special type of exchange economy where it is expected that selling-off firms clear the market,

and we are back to the traditional neoclassical equilibrium. But surprisingly, the model exhibits situations where this does not occur, for profit maximization reasons. Selling-off firms minimize the cost of managing their stock. The value of this stock indicates when the selling-off activity finishes, and likewise when the market clears. Individuals who buy on the selling-off market are assumed not to have any preference for quantity. This is important, since otherwise, there will be no selling-off market at all. Changes in prices are now possible and take place along a modified supply curve (as will become clearer below) and along the traditional demand curve. There is a jump in price that generates a jump in stock. This is new.

We now turn to the presentation of the two models.

1.2.1 Consumer Behavior : the demand side

The objective of this subsection is to set up the simplest possible model capable of showing that a competitive γ -equilibrium exists, i.e. at the end of the market period, the stock of unsold goods generated by the preference for quantity γ is not always zero. Since the economic theory relating to cleared markets is well-known, in this chapter we only investigate equilibria where production exceeds sales. In this version of the model, we suppose there is perfect information and certainty and everyone is perfectly rational. There is a single good and its price is perfectly flexible. We assume that the main consumer has a preference for quantity⁵. Such a reasonable assumption — directly issued from both management and marketing empirical observations — dramatically changes the usual conclusion on the market functioning, as will become clearer below. For that reason, we develop hereafter a new way of reasoning.

Consider a firm that sells a single good to a main consumer. Suppose that on the market this main consumer buys a quantity $\underline{q} \in \mathbb{R}_+^*$ if and only if there exists a quantity of displayed goods $\bar{q} \in \mathbb{R}_+^*$ such that $\bar{q} \geq \underline{q}$. Note that in neoclassical equilibria, market solutions satisfy the clearing market condition if $\bar{q} = \underline{q}$, where the preference for quantity does not hold.

5. There is no range effect or variety effect, since we only consider a single good on the market. Any given individual does not want to buy the last three apples left by the other consumers. He prefers to choose the three apples he wants among the various available apples. Since this is true for all consumers, we capture them in the concept of the main consumer.

We concentrate hereafter on the other set of equilibria where $\bar{q} > \underline{q}$. We show that they exist and study their properties. In these equilibria, if there is an insufficient quantity of displayed goods, no consumer will want to buy anything.⁶ The excess of supply (and not just the supply) creates the demand.

Definition 1. We call the **market period** the traditional period where supply and demand result in a market price (compatible with stocks of unsold goods), before the selling-off period.

Definition 2. We call **selling-off period price** any price that does not clear the market during the market period.

1.2.1.1 The main consumer's behavior

Let α and β be two real numbers such that $0 < \alpha < \beta < \infty$.⁷ By $I := [\alpha, \beta]$ we denote the set of consumption goods, which is a subset of \mathbb{R}_+ . A bounded interval is chosen since the model presented hereafter chapter deals with restricted quantity of goods despite that the main consumer has a preference for quantity.

Suppose that consumer's preferences are represented by the function :

$U : \mathcal{D}(U) \rightarrow [- \infty, \infty)$, such that $(\bar{q}, \underline{q}, \gamma) \mapsto U(\bar{q}, \underline{q}, \gamma)$, where

$$\mathcal{D}(U) = \{(\bar{q}, \underline{q}, \gamma) \in I \times I \times [\underline{\gamma}, \bar{\gamma}] : \bar{q} \geq \underline{q}\}$$

is the domain of U and γ is a parameter which captures the consumer's preference for quantity. Depending on the value of parameter γ we distinguish in this chapter two types of consumers : the main consumer and the residual consumer.

6. No one wants to take the last apple in a basket, which nobody else wanted to choose. Equivalently, no one wants to enter an empty shop.

7. Note that α is analogous to s and β to S in the (s, S) model, and that s exactly corresponds to the empirical concept of security (or safety) stock from the producer's point of view (see James H. Greene (1997)). Since we do not have any uncertainty on the demand side here, we will not use this term in this version of the paper.

Definition 3. We call **main consumer** any consumer who has a positive preference for quantity, i.e. $\underline{\gamma} > 0$. We call residual consumer any consumer who has no preference for quantity, i.e. $\underline{\gamma} = \bar{\gamma} = \gamma = 0$.

Given the characteristics of consumer related to the preference for quantity γ , the set of consumption goods will be defined. Let us denote :

$$\mathcal{D}^\circ(U) = \{(\bar{q}, \underline{q}, \gamma) \in I \times I \times [\underline{\gamma}, \bar{\gamma}] : \bar{q} > \underline{q}\}$$

the interior of $\mathcal{D}(U)$ for the relative topology of $I \times I \times [\underline{\gamma}, \bar{\gamma}]$.

Assumption 1. For all $(\bar{q}, \underline{q}, \gamma) \in \mathcal{D}^\circ(U)$, we have $U(\bar{q}, \underline{q}, \gamma) \in \mathbb{R}$.

If $\bar{q} \in I$, then $\mathcal{D}^\circ(U)_{\bar{q}, \cdot, \gamma} = \{\underline{q} \in I : (\bar{q}, \underline{q}, \gamma) \in \mathcal{D}^\circ(U)\}$,

where

$$\mathcal{D}^\circ(U)_{\bar{q}, \cdot, \gamma} = \begin{cases} [\alpha, \bar{q}) & \text{if } \bar{q} \in (\alpha, \beta], \\ \emptyset & \text{otherwise.} \end{cases}$$

If $\underline{q} \in I$, then $\mathcal{D}^\circ(U)_{\cdot, \underline{q}, \gamma} = \{\bar{q} \in I : (\bar{q}, \underline{q}, \gamma) \in \mathcal{D}^\circ(U)\}$, where

$$\mathcal{D}^\circ(U)_{\cdot, \underline{q}, \gamma} = \begin{cases} (\underline{q}, \beta] & \text{if } \underline{q} \in [\alpha, \beta), \\ \emptyset & \text{otherwise.} \end{cases}$$

Given the preference for quantity γ and a quantity of displayed goods, the first case of Assumption 1 means that the consumer chooses the quantity of goods such that $\bar{q} > \underline{q}$. However, in the second case the main consumer chooses the quantity of goods consumed for a given quantity of displayed compatible with the preference for quantity.

Assumption 2. The function U is continuous on $\mathcal{D}(U)$ and is of class \mathcal{C}^1 on the set $\mathcal{D}^\circ(U)$.

The assumption 2 implies the existence of the utility function U that represents the consumer's preferences and also guarantees a solution for the following consumer's problem.

Given the consumer's income $\Omega \in \mathbb{R}_+^*$ and the price of the good $p \in (0, \infty)$, the rational consumer solves the following problem \mathcal{P} :

$$\mathcal{P} : \begin{cases} \text{Maximize} & U(\bar{q}, \underline{q}, \gamma) \\ \text{w.r.t.} & \underline{q} \in \mathcal{D}(U)_{\bar{q}, \gamma} \\ \text{s.t.} & p\underline{q} \leq \Omega \end{cases}$$

The procedure by which we solve problem \mathcal{P} is as follows. First, we suppose that the consumer's demand for displayed goods $\bar{q} \in (\alpha, \beta]$ is given and we determine his demand for good $\underline{q}^* \in [\alpha, \bar{q})$ with respect to consumer's budget constraint. The consumer is ready to buy such a quantity if and only if his utility reaches a certain level, say $v(\gamma)$. Second, we replace \underline{q}^* in the utility function, knowing that the utility must equate $v(\gamma)$. This helps determine the demand for displayed goods \bar{q} .

The type of utility function we have in mind increases when the quantity of displayed goods \bar{q} increases, see Figures 1 and 2 as well as Section 1.4 for more details. Whatever the utility function, there are two cases : the corner solution where the budget constraint binds and the utility function is irrelevant (interpreted as capturing the rational buyer's behavior), and the interior solution where the utility function is maximized and the budget constraint is irrelevant (interpreted as capturing the impulse buyer's behavior). We now turn to the study of these two cases.

1.2.1.2 The corner solution : the rational buyer

In this paragraph, we introduce the following assumptions for the consumer's problem \mathcal{P} to admit a corner solution.

Assumption 3. Given $(\Omega, p) \in \mathbb{R}_+ \times \mathbb{R}_+^*$, we have $\alpha \leq \frac{\Omega}{p} < \bar{q}$.

Assumption 4. Given Assumptions 1, 2 and 3, for all $\underline{q} \in [\alpha, \frac{\Omega}{p}]$, we have

$$\frac{\partial U(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} > 0.$$

Note that if assumptions 12 and 15 do not hold, then the consumer's problem admits an interior solution. The latter will be discussed in the second case.

1. The demand for good \underline{q}

By Assumption 11, the function $U : \mathcal{D}(U) \mapsto [- \infty, \infty)$ is continuous. By Assumptions 10 and 11, the partial function $U(\bar{q}, \cdot, \gamma) : \mathcal{D}^\circ(U)_{\bar{q}, \cdot, \gamma} \rightarrow (- \infty, \infty)$ is continuous too. Furthermore, by Assumptions 12 and 15, since the restrictive utility function to the closed and bounded interval $[\alpha, \frac{\Omega}{p}] \subset \mathcal{D}^\circ(U)_{\bar{q}, \cdot, \gamma}$ is increasing, then the partial function $U(\bar{q}, \cdot, \gamma)$ reaches its maximum on the compact set $[\alpha, \frac{\Omega}{p}]$. Given the above, there exists an optimal solution \underline{q}^* that satisfies the constrained consumer's problem \mathcal{P} . The quantity of good for which the consumer's utility function is maximized is denoted by :

$$\underline{q}^* = \frac{\Omega}{p}, \quad \text{which will be denoted by } \underline{q}^* := \underline{q}(\Omega, p), \quad (1.1)$$

where p is the market-period price, and the upper-script \star stands for 'corner solution'.

2. The demand for displayed good \bar{q}

The demand for displayed good \bar{q} is obtained as follows. Replacing the consumer's optimal consumption \underline{q}^* in $U(\bar{q}, \cdot, \gamma)$, the consumer's indirect utility function is given by the following expression :

$$U(\bar{q}, \underline{q}^*, \gamma) = v(\gamma). \quad (1.2)$$

For a budget-constrained consumer, the latter expression captures the equation of the highest indifference curve achievable for any given \bar{q} and the preference for quantity γ . In order to guarantee a global result allowing us to determine \bar{q} , it is important to state the following supplementary assumption.

Assumption 5. Given $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ with $\underline{\gamma} > 0$, for all $\underline{q} \in [\alpha, \frac{\Omega}{p}]$, for all $\bar{q} \in (\underline{q}, \beta]$, we have $\frac{\partial U(\bar{q}, \underline{q}, \gamma)}{\partial \bar{q}} > 0$.

We assume that if the main consumer is budget constrained, for all the quantity of goods \underline{q} and for all level of the preference for quantity parameter the consumer's utility increases when the quantity of displayed goods increases. In order to globally

show the existence and the uniqueness of the quantity of the displayed goods in the competitive market it is important to introduce the following lemma.

Lemma 1. Under assumptions 10, 11, 12 and 16, there exists a unique solution to equation (1.2), which is denoted by :

$$\bar{q}^* = h_\epsilon(\Omega, p, v(\gamma)) = h_\epsilon(\underline{q}^*, v(\gamma)) \quad (1.3)$$

Proof. Uniqueness. Given $\underline{q} \in [\alpha, \frac{\Omega}{p}]$, by using the previous assumption we show that the function

$$U(., \underline{q}, \gamma) : (\underline{q}, \beta] \rightarrow (-\infty, \infty), \bar{q} \mapsto U(\bar{q}, \underline{q}, \gamma)$$

is increasing, then it is injective. Denoting the range of the partial function $U(., \underline{q}, \gamma)$ by

$$\mathcal{R}_{\underline{q}, \gamma} = \{U(\bar{q}, \underline{q}, \gamma) : \bar{q} \in (\underline{q}, \beta]\},$$

then

$$U(., \underline{q}, \gamma) : [\underline{q}, \beta] \rightarrow \mathcal{R}_{\underline{q}, \gamma}$$

is obviously a one-to-one function. Since, by Assumption 11, $U(., \underline{q}, \gamma)$ is continuous as the restrictive function U on the compact set $[\underline{q}, \beta]$ and since $\mathcal{R}_{\underline{q}, \gamma}$ is a Hausdorff topological space as topological subset of $[-\infty, \infty]$ which is a topological space, we can conclude that $\bar{q} \mapsto U(\bar{q}, \underline{q}, \gamma)$ is a homeomorphism (Dugundji, Theorem 2.1, p. 226, 1966)⁸. Thus we can globally define the invertible function $\psi_{\underline{q}, \gamma} : \mathcal{R}_{\underline{q}, \gamma} \rightarrow [\underline{q}, \beta]$ of $U(., \underline{q}, \gamma)$ which is automatically continuous.

Let us define the function $v : [\underline{\gamma}, \bar{\gamma}] \rightarrow \mathcal{R}_{\underline{q}, \gamma}, \gamma \mapsto v(\gamma)$ such that for all

$$\gamma \in [\underline{\gamma}, \bar{\gamma}], \psi_{\underline{q}, \gamma}(v(\gamma)) \in [\underline{q}, \beta].$$

8. In topology and related branches of mathematics, a Hausdorff space, separated space or T2 space is a topological space in which distinct points have disjoint neighborhoods. Of the many separation axioms that can be imposed on a topological space, the "Hausdorff condition" (T2) is the most frequently used and discussed.

The latter property ensures that for all level of the consumer's preference for quantity γ , the quantity of displayed goods mentioned above is still higher than the quantity of goods \underline{q} that the main consumer is willing to buy with respect to his budget constraint.

Define the function $\psi : (\underline{q}, v(\gamma)) \mapsto \psi_{\underline{q}, \gamma}(v(\gamma))$ and the interval \mathbb{A}_ε such that :

$$\forall \varepsilon > 0, \quad \mathbb{A}_\varepsilon = \left[U\left(\frac{\Omega}{p} + \varepsilon, \frac{\Omega}{p}, \gamma\right), U\left(\beta, \frac{\Omega}{p}, \gamma\right) \right].$$

Note that \mathbb{A}_ε is the interval of the utility level $v(\gamma)$ such that $\bar{q} \in \left[\frac{\Omega}{p} + \varepsilon, \beta \right]$. The real positive number ε allows us to satisfy the condition for which the quantity of displayed goods \bar{q} is larger than the quantity of goods \underline{q} , and then the preference for quantity parameter is not equal to zero. In particular, this condition guarantees the existence of the log-utility defined in the last subsection of this chapter. But regarding the CES-utility function, the ε value is irrelevant.

Given the result (1.1) of the consumer's problem we have the function h_ε defined by :

$$h_\varepsilon : (\Omega, p, v(\gamma)) \mapsto \psi\left(\frac{\Omega}{p}, v(\gamma)\right).$$

Denoting the domain of function h_ε :

$$\mathcal{D}(h_\varepsilon) = \left\{ (\Omega, p, v(\gamma)) \in \mathbb{R}_+ \times (0, \infty) \times \mathbb{A}_\varepsilon : \frac{\Omega}{p} \in [\alpha, \beta] \right\}$$

and its range by :

$$\mathcal{R}(h_\varepsilon) = \bigcup_{\frac{\Omega}{p} \in [\alpha, \beta]} [\Omega/p, \beta] = [\alpha, \beta].$$

By construction, the function h_ε satisfies the following property :

$$\forall (\Omega, p, v(\gamma)) \in \mathcal{D}(h_\varepsilon), U\left(h_\varepsilon(\Omega, p, v(\gamma)), \frac{\Omega}{p}, \gamma\right) = v(\gamma).$$

Given the main consumer's revenue and the competitive market price, the quantity of displayed goods is chosen with respect to the level of the preference for quantity parameter. These conditions allow to state the following property of the global

uniqueness :

$$\forall(\Omega, p, v(\gamma)) \in \mathcal{D}(h_\epsilon), \forall \bar{q} \in \left(\frac{\Omega}{p}, \beta \right], U(\bar{q}, \frac{\Omega}{p}, \gamma) = v(\gamma).$$

This implies :

$$\bar{q}^* = h_\epsilon(\Omega, p, v(\gamma)). \quad (1.4)$$

Existence. Consider $(\Omega^0, p^0, v^0(\gamma))$ such that $\frac{\Omega^0}{p^0} \in (\alpha, \beta)$ and $v^0(\gamma) \in \overset{\circ}{\mathbb{A}}_\epsilon$, which is the interior set of \mathbb{A}_ϵ .

Define the function $\Psi : \mathcal{D}^0(U)_{.,\underline{q},\gamma} \times \mathbb{R}_+ \times \mathbb{R}_+^* \times \mathbb{A}_\epsilon \rightarrow \mathbb{R}$ such that :

$$\Psi(\bar{q}, \Omega, p, v(\gamma)) := U(\bar{q}, \frac{\Omega}{p}, \gamma) - v(\gamma).$$

Consider $(\Omega^0, p^0, v^0(\gamma)) \in \mathcal{D}^0(U)_{.,\underline{q},\gamma}$ such that

$$\Psi(h_\epsilon(\Omega^0, p^0, v^0(\gamma)), \Omega^0, p^0, v^0(\gamma)) = 0.$$

By Assumption 11, Ψ is of class \mathcal{C}^1 and by Assumption 16, we have

$$\frac{\partial \Psi(h_\epsilon(\Omega^0, p^0, v^0(\gamma)), \Omega^0, p^0, v^0(\gamma))}{\partial \bar{q}} = \frac{\partial U(h_\epsilon(\Omega^0, p^0, v^0(\gamma)), \frac{\Omega^0}{p^0}, \gamma)}{\partial \bar{q}} > 0.$$

Using the Implicit Function Theorem (see H. Cartan, 1977, Theorem 4.7.1., p. 61), there exist a neighborhood \mathcal{V} of $(h(\Omega^0, p^0, v^0(\gamma)), \Omega^0, p^0, v^0(\gamma))$, as well as an open set \mathcal{W} of $(\Omega^0, p^0, v^0(\gamma))$ and a unique function $g : \mathcal{W} \rightarrow \mathbb{R}$ of class \mathcal{C}^1 such that :

- $\forall(\Omega, p, v(\gamma)) \in \mathcal{W}, \Psi(g(\Omega, p, v(\gamma)), \Omega, p, v(\gamma)) = 0,$
- $g(\Omega^0, p^0, v^0(\gamma)) = h_\epsilon(\Omega^0, p^0, v^0(\gamma)),$
- $\{(\bar{q}, \Omega, p, v(\gamma)) \in \mathcal{V} : \Psi(\bar{q}, \Omega, p, v(\gamma)) = 0\}$

$$= \{(g(\Omega, p, v(\gamma)), \Omega, p, v(\gamma)) : (\Omega, p, v(\gamma)) \in \mathcal{W}\}.$$

□

Thus we have shown that the demand function defined in (1.4) exists and is unique. Nothing general can be said about the sign of $\partial \bar{q}^* / \partial \Omega$, $\partial \bar{q}^* / \partial p$ and $\partial \bar{q}^* / \partial \gamma$. Like the examples with usual utility functions (see Section 1.4), various signs are possible. For usual ln and CES utility functions given in Section 1.4, we can prove that the function is decreasing for low p and increasing for high p .

1.2.1.3 The interior solution : the impulse buyer

In this paragraph, we make the following assumptions for a main consumer's interior solution to the problem \mathcal{P} , where the utility function is maximized and the budget constraint is irrelevant. This kind consumer mentioned in the introduction of this chapter is interpreted as capturing the impulse buyer's behavior⁹. Regarding the nature of the main consumer's interior solution problem, it is important to introduce the following assumption.

Assumption 6. The function $U(\bar{q}, \cdot, \gamma)$ is concave on $[\alpha, \min\{\frac{\Omega}{p}, \bar{q}(\gamma)\}]$.

Since the main consumer is not budget constrained, the optimal interior solution may be higher than the optimal solution obtained in the budget constrained case developed in the preceding paragraph.

The non-constrained consumer solves the following problem :

$$\begin{cases} \text{Maximize} & U(\bar{q}, \underline{q}, \gamma) \\ \text{w.r.t.} & \underline{q} \in \bar{\mathcal{D}}(U)_{\bar{q}, \gamma} \end{cases}$$

Lemma 2. Under Assumption 6, there exists a unique optimal demand for good \underline{q} (respectively for displayed good \bar{q}) which is independent of the market price, so that :

$$\underline{q}^{**} = \underline{q}(\bar{q}, \gamma), \tag{1.5}$$

$$\bar{q}^{**} = \ell(v_0(\gamma)). \tag{1.6}$$

9. Cobb and Hoyer (1986) as well as Kollat and Willett (1967) defined the impulse buying as an unplanned purchase. In another research by Rook (1987) underlined that impulse buying usually takes place when a consumer feels a forceful motivation that turns into a desire to purchase a commodity instantly. Beatty and Ferrell (1998) defined impulse buying as instantaneous purchase having no previous aim or objective to purchase the commodity. Stern (1962) found that products bought on impulse are usually cheap. However in this chapter the impulse buyer makes the decision without respect to his budget constraint

Furthermore, if $\frac{\partial^2 U(\bar{q}, \underline{q}, \gamma)}{\partial \bar{q} \partial \underline{q}} > 0$ then the demand for good $\underline{q}(\bar{q}, \gamma)$ is increasing in the demand for displayed good \bar{q} .

Proof. This proof of this lemma is conducted in two stages. First we determine existence and uniqueness of \underline{q} , respectively \bar{q} , and second we characterize the sign of the derivatives.

1. The demand for \underline{q}

The first-order necessary condition of optimality is given by :

$$\frac{\partial U(\bar{q}, \underline{q}^{**}, \gamma)}{\partial \underline{q}} = 0.$$

Since the solution is interior, the latter condition becomes sufficient to allow us to determine the following relation :

$$\underline{q}^{**} = \underline{q}(\bar{q}, \gamma). \tag{1.7}$$

The equation (1.7) implies that the optimal quantity of goods that the consumer buys depends on the preference for quantity γ and the quantity of displayed goods. The latter variable becomes the strategy variable on the market.

We know by Assumption 6 that the second partial derivative $\frac{\partial^2 U(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}^2} < 0$ for all \underline{q} . Thus the following function :

$$U(\bar{q}, \cdot, \gamma) : \underline{q} \mapsto U(\bar{q}, \underline{q}, \gamma)$$

is strictly concave. Consider the function $g : \mathcal{U} \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ such that :

$$g(\underline{q}, \bar{q}, \gamma) = \frac{\partial U(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} = 0,$$

where \mathcal{U} is an open set of \mathbb{R}^2 . Since the function $(\bar{q}, \underline{q}, \gamma) \mapsto U(\bar{q}, \underline{q}, \gamma)$ is of class \mathcal{C}^2 , the function g is of class \mathcal{C}^1 and since we have

$$\frac{\partial g(\bar{q}, \underline{q}^{**}, \gamma)}{\partial \underline{q}} = \frac{\partial^2 U(\bar{q}, \underline{q}^{**}, \gamma)}{\partial \underline{q}^2} < 0,$$

the Implicit Function Theorem can be used. Then there exists an open neighborhood $\mathcal{O}_{\bar{q}, \gamma}$ of (\bar{q}, γ) and a unique function $m : \mathcal{O}_{\bar{q}, \gamma} \rightarrow \mathbb{R}$ of class \mathcal{C}^1 such that the relation (1.5) of Lemma 2 is true.

Remark 1. From the total differential of the equation $g(\bar{q}, \underline{q}(\bar{q}, \gamma)) = 0$, we can obtain the following expression :

$$\frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}} = - \left[\frac{\partial g(\bar{q}, \underline{q}(\bar{q}, \gamma), \gamma)}{\partial \underline{q}} \right]^{-1} \frac{\partial g(\bar{q}, \underline{q}(\bar{q}, \gamma), \gamma)}{\partial \bar{q}},$$

which matches the Marginal Rate of Substitution (MRS). Note that :

$$\frac{\partial g(\bar{q}, \underline{q}(\bar{q}, \gamma), \gamma)}{\partial \bar{q}} > 0 \Rightarrow \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}} > 0. \quad (1.8)$$

2. The demand for \bar{q}

The demand for \bar{q} is obtained as follows :

$$U(\bar{q}, \underline{q}(\bar{q}, \gamma), \gamma) = v_0(\gamma),$$

where the level of indifference curve $v_0(\gamma)$ is obtained by the same procedure in the proof of Lemma 1 which defines the function $v(\gamma)$. Note that these two expressions, $v_0(\gamma)$ and $v(\gamma)$ representing the levels of the main-consumer and residual consumer utility, are not necessarily equal. For a non-constrained consumer, the above expression captures the equation of the highest indifference curve achievable for any given \bar{q} and the preference for quantity γ .

Define the function ζ by

$$\zeta(\bar{q}, v_0(\gamma)) = U(\bar{q}, \underline{q}(\bar{q}, \gamma), \gamma) - v_0(\gamma).$$

Consider $(\bar{q}, v_0(\gamma))$ such that

$$\zeta(\bar{q}, v_0(\gamma)) = 0.$$

Suppose that the partial derivative $\frac{\partial \zeta(\bar{q}, v_0(\gamma))}{\partial \bar{q}} \neq 0$. As in the lemma 1, by the Implicit Function Theorem, there exist a neighborhood of $v_0(\gamma)$ and a unique continuous function ℓ such that we have :

$$\bar{q}^{**} = \ell(v_0(\gamma)). \quad (1.9)$$

Relations (1.5) and (1.6) are consistent with the marketing literature that estimates the demand function without taking explicitly into account the price level as an explicative variable. In this literature, authors consider that demand is price-independent and displayed good-dependent. For that reason, we label this consumer's behavior as impulse buying. Note that price plays a role in equilibrium, since the firm's profit is price-dependent. In this context, many studies have used impulse buying (or purchasing) to view the determinant of consumer behavior. Some authors have measured the incidence of impulse purchasing and have shown how different kinds of products are affected by it, (Kollat and Willett (1967)). Since our work does not take into account of variety effect, we insist only on the preference for quantity γ and the quantity of displayed goods as factors that influence the main non-constrained consumer behavior.

3. The signs of the derivatives

We now show that $q(\bar{q}, \gamma)$ is increasing in \bar{q} . As a result of the Implicit Function Theorem stated in the first relation of the Lamma, we have the following expression :

$$\frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} = - \left[\frac{\partial g(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} \right]^{-1} \frac{\partial g(\bar{q}, \underline{q}, \gamma)}{\partial \bar{q}}.$$

Since by Assumption 6, we have :

$$- \left[\frac{\partial g(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} \right]^{-1} = - \left[\frac{\partial^2 U(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}^2} \right]^{-1} > 0,$$

we must necessarily have

$$\frac{\partial g(\bar{q}, \underline{q}, \gamma)}{\partial \bar{q}} > 0$$

which achieves the proof. □

This result confirms through the signs of the derivatives the idea that the quantity of quantity goods has an impact on the consumer behavior.

1.2.1.4 The residual consumer

This subsection deals with the residual consumer we have already mentioned in the first section of the chapter. We suppose that the residual-consumer buys a quantity $\tilde{q} \in \mathbb{R}_+^*$ in the selling-off market. According to the definition (4) the parameter of the preference for quantity γ is henceforth equal to zero.

The new problem of the residual consumer is to maximize his utility, $\tilde{U} : [\alpha, \beta) \rightarrow \mathbb{R}$ subject to his budget constraint :

$$\begin{cases} \text{Maximize} & \tilde{U}(\tilde{q}_c) \\ \text{s.t.} & \tilde{p}\tilde{q} = \Omega_r, \end{cases}$$

where \tilde{p} is the selling-off price, and $\Omega_r < \Omega$ is the income of the residual consumer. We assume that the utility function \tilde{U} of the residual consumer is increasing, concave and continuously differentiable. Furthermore, since the budget constraint is bounded, the solution of the consumer's problem is given by :

$$\tilde{q}_k^* = \frac{\Omega_r}{\tilde{p}_k}. \quad (1.10)$$

1.2.2 Competition

In this subsection, we consider a competitive market structure. The demand $D(p)$ may come from a constrained or unconstrained consumer (see \mathcal{P}). We shall focus on the two following cases. The first is devoted to the producer's problem with respect to the constrained consumer while the second is devoted to the unconstrained consumer. The profit of the producer is formulated by :

$$\pi = pD(p) - TC(\bar{q}), \quad (1.11)$$

where $D(p)$ stands for the consumer's demand for good q , which is detailed below. This function is very close in spirit to the one proposed by Scarf (2002), except that here the demand for good q is not a random variable, but a deterministic one.¹⁰

10. In his paper, Scarf considers the following profit function :

$$r \min\{y, \xi\} - cy$$

Theorem 1. In the competitive market structure, there exists a neoclassical market equilibrium such that the price leads the economy to a γ -equilibrium. Production always exceeds sales. Production and sales co-move.

Proof. The proof is in two parts. The first is devoted to the competitive market equilibrium respect to the consumer's corner solution and the second to the interior solution.

1.2.2.1 The competitive producer : the supply side

We now study the centrality of the neoclassical paradigm : perfect competition. In our neoclassical theory, we keep all the traditional assumptions and add the preference for quantity. As the pure competitive market structure suggests, we assume atomistic producers and consumers, product homogeneity, and perfect information. There are no barriers to entry or exit in the long run (zero-profit condition). As a consequence, in equilibrium the demand curve is perfectly elastic. Price-taking main consumers buy a utility-maximizing quantity of goods at the prevailing price. Similarly, each price-taking producer sells its profit-maximizing quantity at the same prevailing price.

By the assumption of atomistic consumers and firms, no single agent can influence the market price. In the short run, the market sets the price and each producer reacts to that price by manipulating the strategic variables. In this case, the demand curve takes the following form :

$$D(p) = \begin{cases} \bar{q}^{s*} & \text{if } p < p_\gamma \text{ and } \bar{q}^{s*} \geq \underline{q}^* & (1) \\ \underline{q}^* & \text{if } p = p_\gamma \text{ and } \bar{q}^{s*} \geq \underline{q}^* & (2) \\ 0 & \text{if } p > p_\gamma, \forall \bar{q} & (3) \end{cases}$$

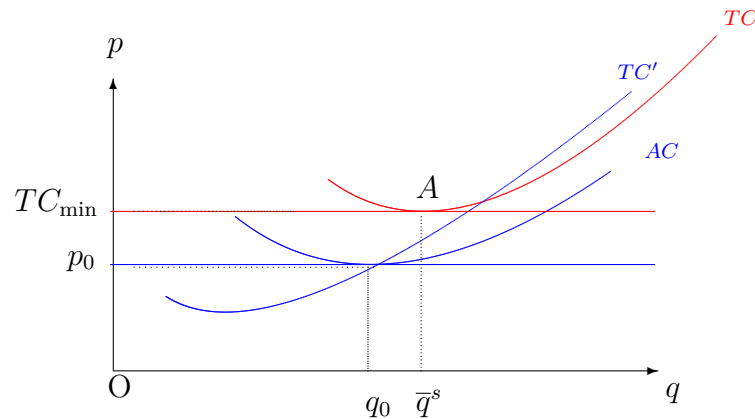
where \bar{q}^{s*} is the profit-maximizing quantity of displayed goods, \underline{q}^* the utility-maximizing quantity of goods, and p_γ the γ -equilibrium price. It is interesting to note that :

1) the case (1) exactly describes the traditional competitive model, where the only possible price $p < p_\gamma$ is the competitive market price.

b) the case (2) extends the case (1) and allows for the emergence of a stock of unsold goods.

where r is the price of the good, y is the available quantity and ξ is a random variable capturing the demand. The storage cost is linear. Note that such a model cannot easily reproduce any co-movement between production and sales.

FIGURE 1.1 – Standard Equilibrium and γ -equilibrium : Case 1



In blue, the standard competitive equilibrium

c) and the case (3) as usual means that if the competitive firm sets a price above the prevailing market price, neither the main-consumer nor the residual consumer will purchase its product.

In red, the supplied quantity of displayed goods minimizes the total cost

Furthermore, note that in the first case the consumer has the opportunity to choose the quantity of goods supplied by the firm despite his demand for displayed is \bar{q}^* . Contrary to Varian (1992) we consider that the consumer chooses the quantity of goods \bar{q}^{s*} which is higher than the demand for \underline{q}^* .

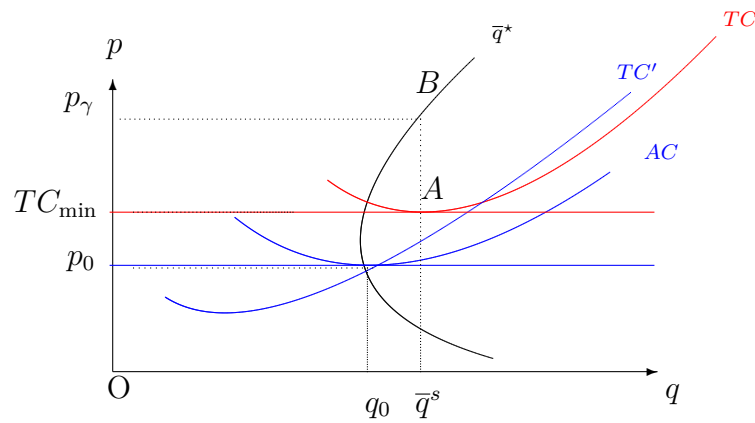
1.2.2.2 Corner solution

In the short run, the rational producer always maximizes its profit. It turns out that for the consumer's demand for the good, previously called corner solution :

$$\underline{q}^* = \Omega/p,$$

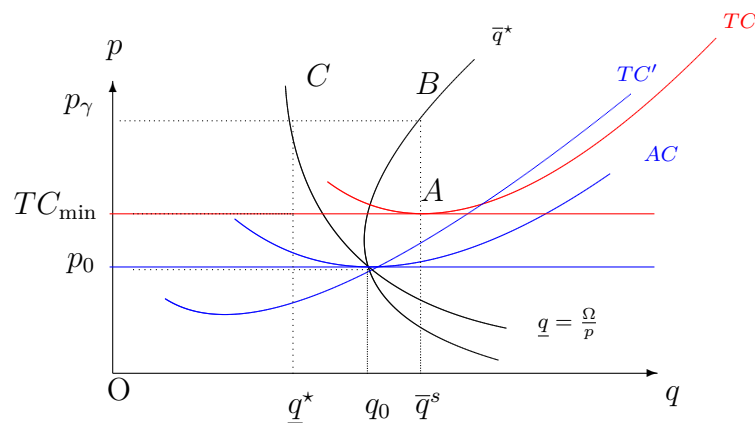
the problem of the competitive profit-maximizing firm is equivalent to minimizing the production costs, which is much simpler to solve (see Section 4). The following figures show how the equilibrium is reached.

FIGURE 1.2 – Standard Equilibrium and γ -equilibrium : Case 2



The demand for displayed goods equals the optimal output at p_γ for corner solution, and $U_1 = \ln$ or $U_2 = CES$ utility functions, (see Section 4).

FIGURE 1.3 – Standard Equilibrium and γ -equilibrium : Case 3



The demand for goods crosses the price p_γ , which gives \underline{q}^* for corner solution, and $U_1 = \ln$ or $U_2 = CES$ utility functions, (see Section 4).

$\pi = 0$ means $\Omega = TC$ or equivalently surface $OTC_{\min}A\bar{q}^s = Op_\gamma C\bar{q}^*$.

The volatility of the production \bar{q} is higher than the volatility of sales \underline{q} in the neighborhood of the γ -equilibrium.

In Figure 1, variables have been chosen so that the quantity of displayed goods is greater than the optimal quantity q_0 that would have been produced in the traditional competitive equilibrium. Obviously, it may happen that this relation is reversed. Note that in Figure 3, for any price increase in the neighborhood of the competitive γ -equilibrium, the demand for displayed goods (\bar{q}) grows faster than the demand of goods \underline{q} decreases. This explains why production volatility is higher than sales.

Contrary to the traditional model, even if price is flexible and moves in the direction of the excess demand, it sticks to the competitive γ -equilibrium, without reaching the traditional competitive equilibrium. Consequently, in our model, one has to consider the stock of displayed goods, demand for goods and prices as indexes of the consumer's welfare. Moreover, no government policy to restore the traditional competitive equilibrium through optimal taxation can be implemented here, since the competitive γ -equilibrium is not due to a market failure in this model. It is generated by the preference for quantity principle, so that no one can prevent individuals from following their own preferences. Thus three previous figures show that in a frictionless model of competitive market, the preference for quantity generates a γ -equilibrium.

1.2.2.3 Interior solution

We have shown in the preceding case for which the consumer is not budget constrained, that a competitive γ -equilibrium is generated in the frictionless model of competitive market. However, in this subsection we consider the case of an interior solution stated in (1.5) for the competitive firm's problem. The profit maximization problem for such a price-taking firm could be written as follows :

$$\begin{aligned} & \text{Maximize } p\underline{q}(\bar{q}, v(\gamma)) - TC(\bar{q}). \\ & \text{w.r.t. } \bar{q} \end{aligned}$$

Given the market price p , the first-order and second-order conditions are given respectively by :

$$p = \left(\frac{\partial \underline{q}(\bar{q}, v(\gamma))}{\partial \bar{q}} \right)^{-1} TC'(\bar{q}), \tag{1.12}$$

$$\partial^2 \pi / \partial \bar{q}^2 \leq 0.$$

An important remark should be made here. The new competitive and profit-maximizing price (1.12) differs from the usual competitive pricing. Usually, price equals marginal cost, but not here. The new competitive price is greater than the traditional one as long as we have the following inequality :

$$\partial \underline{q} / \partial \bar{q} < 1.$$

Since it is commonly observed that production is more volatile than sales, we assume that $\partial \underline{q} / \partial \bar{q} < 1$. Obviously, from a pure theoretical point of view, the reverse relation may also hold. Consequently, the above competitive price (1.12) is greater than the traditional competitive price. To better appreciate the difference, let us first define the ratio of displayed goods over the demand for goods, $r = \bar{q} / \underline{q}$, and rewrite the first order condition (1.12) as follows :

$$\begin{cases} p = r \frac{\varepsilon_{TC/\bar{q}}}{\varepsilon_{\underline{q}/\bar{q}}} AC \\ r \varepsilon_{TC/\bar{q}} > \varepsilon_{\underline{q}/\bar{q}}. \end{cases}$$

where AC is the average cost, $\varepsilon_{TC/\bar{q}}$ is the elasticity of the total cost with respect to the quantity of displayed goods and $\varepsilon_{\underline{q}/\bar{q}}$ is the elasticity of the displayed goods with respect to the demand. Note that in the traditional competitive equilibrium we have

$$\varepsilon_{TC/\bar{q}} = r \varepsilon_{\underline{q}/\bar{q}}.$$

At the prevailing price p , the solution of this maximization problem tells us what quantity of \bar{q} the firm will produce and display in order to match the main consumers' demand for displayed goods, knowing that their preference for quantity γ stimulates their demand for goods $\underline{q}(\bar{q}, v(\gamma))$. The first-order condition is not really the same as the "marginal revenue = marginal cost" condition associated with the traditional market equilibrium, since the marginal cost is now divided by the sensitivity of the demand for goods relative to the quantity of displayed goods.

The second-order condition for profit maximization is that $\partial^2 \pi / \partial \bar{q}^2 \leq 0$. The marginal profit at \bar{q}^{s*} is zero. Taken together, these two conditions determine the new supply function

of the competitive firm, when preference for quantity is operating. At any price p , the firm supplies an amount of output \bar{q}^{s*} such that p satisfies the condition (1.12) and $\partial^2\pi/\partial\bar{q}^2 \leq 0$. Conversely, in order to induce a competitive firm to supply an amount of output \bar{q}^{s*} , the market price must be that of (1.12) and furthermore $\partial^2\pi/\partial\bar{q}^2 \leq 0$.

Let $p(\bar{q})$ be the inverse supply function, measuring the prevailing price at which the firm finds it profitable to produce a given quantity of goods. The first-order condition for the profit maximization problem can be written as :

$$p(\bar{q}) = \left(\frac{\partial \underline{q}(\bar{q}, v(\gamma))}{\partial \bar{q}} \right)^{-1} TC'(\bar{q}) \quad (1.13)$$

$$TC''(\bar{q}) > 0.$$

Since the supply function $\bar{q}(p)$ gives the profit-maximizing quantity of goods \bar{q} at each price, it must verify the first-order condition. This is due to the fact that the direct supply curve and the inverse supply curve measure the same relationship between price and the profit-maximizing supply of output or quantity of displayed goods.

Hence, we have the following expression which is written with respect to the market price :

$$p = \left(\frac{\partial \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q}} \right)^{-1} TC'(\bar{q}(p)) \quad (1.14)$$

$$TC''(\bar{q}(p)) \geq 0.$$

To see how the competitive firm's supply reacts to a change in the prevailing price p , we differentiate the previous expression with respect to the price p :

$$\left(\frac{\partial \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q}} \right)^2 = \bar{q}'(p) \left(TC''(\bar{q}(p)) \frac{\partial \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q}} - TC'(\bar{q}(p)) \frac{\partial^2 \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q} \partial p} \right).$$

Since the left-hand side of the above relation is positive for all the output level and the consumer utility level, in order for the slope $\bar{q}'(p)$ of the supply curve to be positive, the following condition must hold :

$$\frac{TC''(\bar{q}(p))}{TC'(\bar{q}(p))} \geq \left(\frac{\partial^2 \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q} \partial p} \right) \left(\frac{\partial \underline{q}(\bar{q}(p), v(\gamma))}{\partial \bar{q}} \right)^{-1}. \quad (1.15)$$

Under the previous condition, the firm supply function is increasing with respect to the competitive market price. Clearly the quantity of displayed goods that the competitive firm is willing to produce depends on the market price.

Let us write the total cost function as the sum of variable costs C_v plus fixed costs C_F :

$$TC(\bar{q}) = C_v(\bar{q}) + C_F.$$

The new short-term supply curve is determined as follows. The rational competitive firm decides whether it is more profitable to produce a positive level of output with a fixed cost or not to produce anything, just paying the fixed cost. We have :

$$p\underline{q}(\bar{q}, v(\gamma)) - C_v(\bar{q}) - C_F \geq -C_F.$$

From a simple algebraic transformation, the previous inequality can be rewritten as :

$$p\underline{q}(\bar{q}, v(\gamma)) \geq C_v(\bar{q})$$

or

$$p \geq \frac{C_v(\bar{q})}{\underline{q}(\bar{q}, v(\gamma))}. \quad (1.16)$$

Since $\bar{q} \neq 0$, the latter inequality can be written as :

$$p \geq rVAC(\bar{q}), \quad (1.17)$$

This condition states that the prevailing price must be weakly greater than the variable average cost weighted by the rate at which the displayed goods return. Equation (1.17) allows us to capture the modified supply curve. One can easily show that this curve is above the traditional VAC curve. \square

Lemma 3. $\left(\frac{\partial q(\bar{q}, v(\gamma))}{\partial \bar{q}}\right)^{-1} TC''(\bar{q})$ crosses the minimum of $rVAC(\bar{q})$.

The lemma 3 implies that the prevailing competitive market price crosses the minimum of the variable average cost weighted by the displayed of goods-sales ratio. Proof is straightforward and left to the reader.

One can easily construct the market equilibrium by aggregating all the supplies and all the demands. The traditional equilibrium price ensures that all firms sell their entire output, or equivalently, that total production matches total demand. In the new equilibrium, the price is compatible with the free entry condition and all firms make zero profit.

Lemma 4. Firms cease to enter the market if and only if the long-term market price is

$$p = \frac{TC}{\underline{q}} = rAC, \quad \text{and equivalently the ratio } \varepsilon_{TC/\bar{q}}/\varepsilon_{\underline{q}/\bar{q}} = 1.$$

Proof is simple and left to the reader.

In this case, the return to producing displayed goods is simply the appreciation of the ratio of displayed goods to the final demand adjusted by average cost.

1.2.3 The Competitive γ -equilibrium

We now calculate the resulting optimal competitive quantity of unsold goods denoted by S_c . To do so it is important in this chapter to say what we exactly mean by the " **γ -equilibrium**", which is a new concept of equilibrium.

Definition 4. We call " **γ -equilibrium**" any situation where the preference for quantity γ is compatible with an optimal non-zero stock of unsold goods at the end of the market period. In other words, production exceeds sales.

Theorem 2. Given that consumers have a preference for quantity, under pure competition with perfect information, price flexibility, no uncertainty and full rationality, we have the following results :

1. If the consumer is budget-constrained, then there exists a non-zero stock of unsold goods at the market competitive γ -equilibrium price, or there may (or not) exist a non-optimal competitive γ -equilibrium price that clears the stock of unsold goods.
2. If the consumer is not budget-constrained, then the resulting non-zero stock of unsold goods is totally independent of any price, or equivalently there is no price system that clears the stock of unsold goods.

Proof. Dealing with the corner solution, by using the relation (1.1) and the result (1.3) of Lemma 1, we obtain the following optimal stock of unsold goods :

$$S^* = h_\epsilon(\Omega, p, v(\gamma)) - \frac{\Omega}{p} = S(\Omega, p, v(\gamma)). \quad (1.18)$$

As long as the parameter $\gamma \neq 0$ of the preference for quantity, there exists a non-optimal competitive selling-off price that clears the stock of unsold goods. Production may exceed sales.

Given the interior solution, by using relations (1.5) and (1.6) of Lemma 2 the stock unsold goods is determined as follows :

$$S^{**} = \bar{q}^{**} - \underline{q}(\bar{q}^{**}, \gamma) \iff S^{**} = \ell(v_0(\gamma)) - \underline{q}(\ell(v_0(\gamma)), \gamma) = S(v_0(\gamma)). \quad (1.19)$$

Generically, we have $S(v_0(\gamma)) \neq 0$ and independent of prices. Production always exceeds sales. □

As long as the consumer has a preference for quantity, given the expressions (1.18) and (1.19) for the optimal stock of unsold goods, there are three main cases.

1. If the consumer is budget-constrained, then it may be the case that the firm makes a donation of the stock of unsold goods.
2. If the consumer is budget-constrained, then it may be the case that the firm sells the stock of unsold goods at a γ -equilibrium price.

3. If the consumer is not budget-constrained, the firm has an incentive to destroy (or recycle) the stock of unsold goods.

As long as the quantity of displayed goods q is different from the quantity of goods \bar{q} obviously we have $r \neq 1$. Hence, the long-run price $p = rAC$ related to the assumption of preference for quantity differs from the traditional long-run competitive equilibrium $p = AC$ where firms operate at a price such that they can sell their entire output. No dead stock at all is possible in that case.

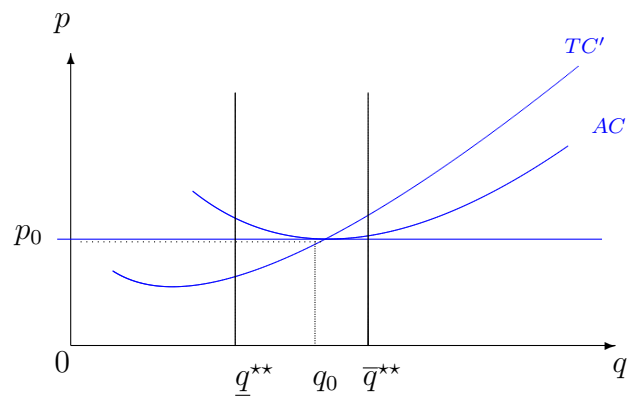
In our model, the situation is quite different, since $r > 1$ by definition, the long-run price is above the traditional price. The interesting property we exhibit now is that even if a firm wants to clear its stock of unsold goods, this cannot be achieved with the long-run price. By reducing the long-run price, one can reach the traditional clearing market price, but this price is not optimal. That is why a selling-off market may play an important role in the economy, since we assume that a selling-off firm has no production cost, but only a storage cost, such that a clearing activity may reduce or even better clear the stock of unsold goods, as will be demonstrated below.

More importantly, when preference for quantity is operating on goods markets during the market period, there is no place for government intervention, since it is futile to fight against individuals' preferences. But from an empirical perspective, on the selling-off market, the government may help to organize the functioning of the market. More precisely, since the optimal price prevailing on the selling-off market does not clear the market, and since there exists a clearing market price, the government could act as a market clearer.

Note that in this new γ -equilibrium price, stocks of unsold goods are positive. We show that the new price is higher than the traditional competitive market equilibrium price, since here we have the demand for goods is lower than the demand of displayed goods. It can also be observed that the two demand curves cross in the traditional competitive equilibrium, but never outside of it.

The next figures sum up our results. The traditional competitive equilibrium is in blue, while the new competitive equilibrium is in red. As long as the parameter value of the

FIGURE 1.4 – Construction of the γ -equilibrium : Case 1



The demand for goods and the demand for displayed goods are exogenous, whatever p_γ is. For interior solution, and $U_1 = \ln$ or $U_2 = CES$ utility functions, see Section 4. In blue : a traditional competitive equilibrium.

preference for quantity $\gamma > 0$, there is no means to reach the traditional competitive equilibrium. There are persistent stocks of unsold goods. The next three figures illustrate the construction of the γ -equilibrium.

FIGURE 1.5 – Construction of the γ -equilibrium : Case 2

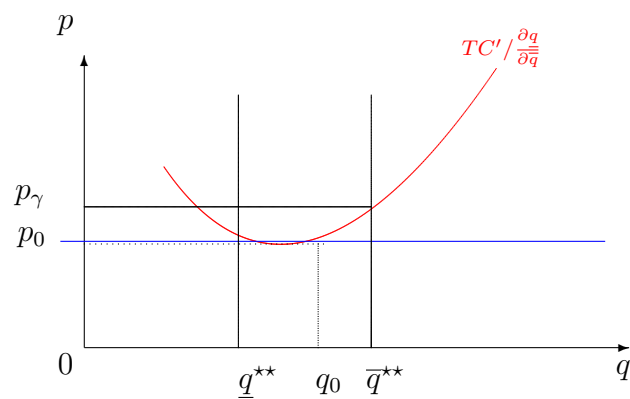


Figure 5 : The supply of goods crosses the demand for displayed goods, giving the price p_γ .

FIGURE 1.6 – Construction of the γ -equilibrium : Case 3

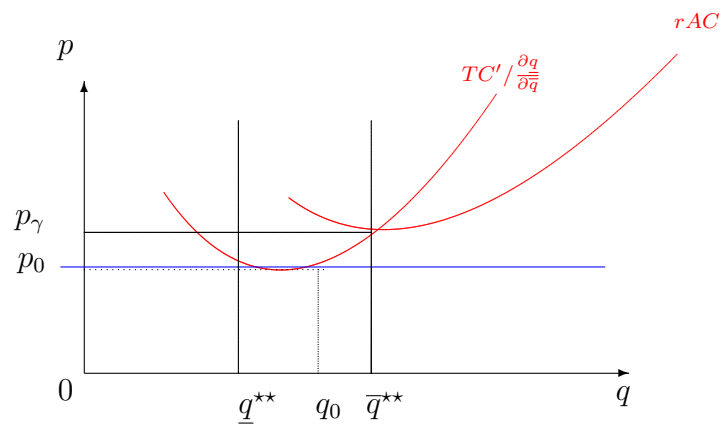


Figure 6 : In the long run, the market γ -equilibrium is such that $p_\gamma = rAC$.

Figure 4 shows the traditional competitive equilibrium for which the quantity of displayed goods \bar{q} and the quantity of goods \underline{q} match. Note that in Figure 6, production co-moves with sales only for a change in the parameter level γ . The demand for goods is lower than the demand for displayed goods by a factor r . A change in price can only be due to some technological improvement. The consumer being insensitive to price, the two demands are rigid. Contrary to the traditional model, since the two demands are totally inelastic to price, price is not moving in the direction of the excess demand. Hereafter we study the functioning of the competitive selling-off market.

1.2.4 The functioning of the competitive selling-off market

In this subsection, we keep the same set of assumptions as above, except that the residual consumer has no preference for quantity. Consequently, the residual consumer is always budget-constrained. Suppose for now that the producer we studied above chooses to delegate the selling-off activity to an external firm, hereafter called the selling-off firm. Let us denote this stock by $S^k = \bar{q}^k - \underline{q}^k$, where $k = \star, \star\star$ stand for the corner and interior solutions respectively. Note that $S^\star = S(\Omega, p, v(\gamma))$ and $S^{\star\star} = S(v_0(\gamma))$.

Proposition 1. Since the residual consumer has no preference for quantity, a non zero-stock competitive selling-off equilibrium exists and selling-off clearing is possible. There are two main cases :

1. The demand of the residual consumer is less than the stock of unsold goods S^k and the market still has a stock of unsold goods, called a dead stock.
2. The demand of the residual consumer is greater or equal to the stock of unsold goods S^k , and the market clears, but this is not an equilibrium.

Proof. Since the residual consumer does not have any preference for quantity, i.e $\gamma = 0$, the demand facing the competitive selling-off firm (index *sof*) is the following :

$$D_{sof} = \begin{cases} 0 & \text{if } p > \tilde{p}_c \quad \forall \tilde{q}_c \\ \tilde{q}^\star & \text{if } p = \tilde{p}_c \quad \text{and} \quad \tilde{q}_c \geq \tilde{q}_c^\star \\ S^k & \text{if } p < \tilde{p}_c \quad \text{and} \quad S^k \leq \tilde{q}_c^\star \end{cases}$$

Contrary to the previous case where the competitive firm faced the main-consumer's preference for quantity on the period market, the only cost incurred by the selling-off firm in this subsection is a storage cost, denoted by $SC(\cdot)$. Thus the profit function of the selling-off firm is :

$$\pi_{sof} = \tilde{p}_c \tilde{q}_c - SC(S^k).$$

On this new type of market, a rational selling-off firm chooses to display the quantity $\tilde{q}_c = S^k$ so as to maximize its profit :

$$\max_{\tilde{q}_c} \tilde{p}_c \tilde{q}_c - SC(\tilde{q}_c).$$

where \tilde{q}_c is the quantity of displayed goods. Exactly as during the market period, we now compute the competitive selling-off firm's solution.

1. The competitive selling-off firm. In the short run, the competitive firm maximizes its profit for any given price \tilde{p} . The first-order condition is the following :

$$\tilde{p} = \frac{dSC(\tilde{q}_c)}{d\tilde{q}_c}.$$

The supply curve is as usual (cf Subsection 1.2.1.), $\tilde{p} \geq VAC$. In the long run, the free entry condition implies the zero profit condition. Equivalently we have :

$$p_c = \frac{SC(\tilde{q}^s)}{\tilde{q}^s} \iff \tilde{q}^s = q^s(p_c). \quad (1.20)$$

One of the crucial features of the competitive selling-off market in the of long run is that the demand of displayed goods faced by the firm is perfectly elastic with respect to the competitive market price. This fact is in line with the traditional competitive market. As usual, the price is equal to the average cost. For an equilibrium to exist, it must be the case that the residual consumer's demand for goods (1.10) matches the demand of displayed goods of the competitive maximizing-profit selling-off firm (1.20). This leads to the following competitive market price :

$$\exists \tilde{p}_c^* \in \mathbb{R}_+ \text{ such that } \tilde{q}_c^* = \tilde{q}^s \iff \frac{\Omega_r}{\tilde{p}_c^*} = q^s(\tilde{p}_c^*),$$

so that

$$\tilde{p}_c^* = p(\Omega_r).$$

The selling-off market is constructed by aggregating all the stocks of unsold goods issued from the identical J firms operating during the market period. For simplicity, we assume that each selling-off firm is connected with one of the J previous firms, so that there are J selling-off firms. It would not change anything if we considered a different selling-off market configuration. We also assume that there are N identical atomistic consumers on the selling-off market.

Definition 5. If there exists $\tilde{p}_{som} \in \mathbb{R}_+^*$ such that $\sum_{j=1}^J S_j^k - \sum_{i=1}^N \tilde{q}_{ic}^*(\tilde{p}_{som}) = 0$, then \tilde{p}_{som} is called a competitive selling-off market equilibrium price.

There is no reason for the market-clearing condition to hold, and we have :

$$\sum_{j=1}^J S_j^k - \sum_{i=1}^N \tilde{q}_{ic}^*(\tilde{p}_{som}) \neq 0,$$

A residual stock of unsold goods remains on the market. Since all firms are identical, and since all consumers are identical too, the market clears if and only if the following condition is satisfied :

$$N\tilde{q}^*(\Omega, p_{som}) = JS^k.$$

This condition specifies the new price that clears the market. This new price is not necessary compatible with the one that the rational selling-off firm optimally chooses.

□

1.3 Alternative Modeling

In this section, we now extend our previous simple approach in two directions. First, we consider that the consumer can either be a main consumer on one good and a residual consumer on the other good (see Brown and Tucker (1961)). These two authors postulated three classes of products with respect to space elasticities : "unresponsive products", "general use products" and "occasional purchase products" (impulse buys). Following them, we

assume that preference for quantity may play a role for some products but not for others. Second, we allow the consumer to maximize his surplus.

1.3.1 The case of two goods

We now extend our previous simple approach to the case of two goods. Let us consider the case where the consumer consumes two consumption goods $(x, \underline{q}) \in I^2 \subset \mathbb{R}_+^2$. The assumption now is that the rational consumer has no preference for quantity on x , but has γ on \underline{q} . Assuming quasi-linear preferences, we solve the following problem :

$$\mathcal{P}_2 \left\{ \begin{array}{l} \text{maximize} \quad x + u(\bar{q}, \underline{q}, \gamma), \\ \text{w.r.t} \quad (x, \underline{q}) \in I \times \mathcal{D}(U)_{\bar{q}, \gamma}, \\ \text{s.t.} \quad x + p\underline{q} \leq \Omega, \\ \quad \quad \underline{q} \leq \bar{q}, \\ \quad \quad \underline{q} > 0, \bar{q} > 0, \quad x > 0. \end{array} \right.$$

Assumption 7. The utility function U is increasing concave and continuously differentiable on the consumption set $\mathcal{D}(U)_{\bar{q}, \gamma}$.

Proposition 2.

1. Assuming a regular point (x, \underline{q}) satisfying the Karush-Khun-Tucker (KKT) conditions , the problem \mathcal{P}_2 has a unique solution denoted by :

$$\underline{q} = g_\ell(\bar{q}, p, \gamma) \tag{1.21}$$

2. Given the consumer's solution, the optimal solution of the competitive firm is expressed as follows :

$$\bar{q}_C = h_C(p, \gamma)\bar{q}_M = h_M(p, \gamma) \tag{1.22}$$

Consequently, assuming quasi-linear preferences and perfect competition, there exists a γ -equilibrium for which (x, \underline{q}) is a non-zero solution of problem \mathcal{P}_2 such that $\underline{q} < \bar{q}$.

Proof. The proof is conducted into two parts. First, we solve the consumer's problem by the KKT conditions. Second, given the consumer's problem solution of the demand for goods the competitive firm chooses the quantity of displayed goods that maximizes its profit function.

The Consumer's Problem

Suppose that (x, \underline{q}) is a regular solution of the problem \mathcal{P}_2 . The \mathcal{L} agrangian of the problem \mathcal{P}_2 is :

$$\mathcal{L}(x, \bar{q}, \underline{q}, \lambda, \mu) = x + u(\bar{q}, \underline{q}, \gamma) + \lambda [\Omega - x - p\underline{q}] + \mu [\bar{q} - \underline{q}].$$

The Karush-Kuhn-Tucker (KKT) conditions are given by :

$$\lambda \geq 0, \quad \mu \geq 0,$$

$$\Omega - x + p\underline{q} \geq 0, \quad \bar{q} - \underline{q} \geq 0,$$

$$\lambda(\Omega - x + p\underline{q}) = 0,$$

$$\mu(\underline{q} - \bar{q}) = 0,$$

$$1 - \lambda = 0,$$

$$\frac{\partial u(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} - \lambda p - \mu = 0.$$

Let us check for all possibilities of active constraints. Since $\lambda = 1$, we have

$$x + p\underline{q} = \Omega.$$

We necessarily have two cases.

If $\mu = 0$, then the constraint is not active, that is : $\bar{q} - \underline{q} \geq 0$. In this case, we have :

$$\frac{\partial u(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} = p. \tag{1.23}$$

The price is equal to the marginal utility with respect to \underline{q} . By using the Implicit Function Theorem, the above equation allows us to determine the following demand functions of the consumer :

$$\underline{q}^* = q_\ell(\bar{q}, p, \gamma), \quad (1.24)$$

$$x^* = \Omega - pq_\ell(\bar{q}, p, \gamma). \quad (1.25)$$

Since the constraint $\bar{q} - \underline{q} \geq 0$ is not active, the consumer's demand satisfies the following condition for which the quantity of displayed goods higher the demand for goods :

$$q_\ell(\bar{q}, p, \gamma) < \bar{q}.$$

If $\mu \neq 0$ and $\underline{q} = \bar{q}$, then the constraint is active. In order for $\underline{q} = \bar{q}$ and $x = \Omega - p\bar{q}$ to be a feasible solution of \mathcal{P}_2 , we must have the following condition :

$$\frac{\partial u(\bar{q}, \bar{q}, \gamma)}{\partial \underline{q}} \geq p. \quad (1.26)$$

If this condition may hold in certain case, regarding our preference for quantity principle it has to be rejected even if it is compatible with the traditional competitive equilibrium.

The Competitive Producer

At the prevailing market price, the competitive firm's profit is given by :

$$\pi_c = p\underline{q} - TC(\bar{q})$$

Dealing with the long term and replacing the consumer's demand in the profit function, we have :

$$pq_\ell(\bar{q}, p, \gamma) - TC(\bar{q}) = 0,$$

which yields the following quantity of goods faced by the competitive firm :

$$\bar{q}_C = g_\ell(p, \gamma). \quad (1.27)$$

In the competitive market, the quantity of goods \bar{q} that the producer is willing to produce depends on the competitive price p and the consumer's preference for quantity

γ . Note that $\underline{q}_\ell \neq \bar{q}_\ell$ is possible. Consider the following example of the consumer's utility function :

$$U(\bar{q}, \underline{q}, \gamma) = x + \frac{1}{2} \left[1 - (1 - \underline{q})^2 + \gamma(\bar{q} - \underline{q})^2 \right].$$

The consumer's solution is :

$$\underline{q}^* = \frac{1 + \gamma\bar{q}}{1 + \gamma}.$$

Generically, $\underline{q}_\ell \neq \bar{q}_\ell$. Many examples will be discussed in the following section. □

1.4 Applications with usual utility functions

The aim of this section is to provide not only the consumer's demands and the producer's prices by using the usual utility function examples, but also the optimal stocks. In the last subsection, different cases will be discussed depending on a competitive context. As previously noted in this paper, index 1 captures examples of the log-utility function, while index 2 captures those of the CES-utility function. We have chosen to illustrate our general developments with these two functions, because they exhibit very different and interesting results, as will be shown below.

1.4.1 Preferences

In this subsection, individuals' preferences are captured with a separable utility function : the log-utility function and the CES-utility function.

Let us first consider the log-utility function.

$$U_1(\bar{q}, \underline{q}, \gamma) = \ln \underline{q} + \gamma \ln(\bar{q} - \underline{q}).$$

Since the consumer's preferences are invariant with respect to any monotonic transformations of utility, the previous function also encompasses the traditional Cobb-Douglas utility function :

$$W_1(\bar{q}, \underline{q}, \gamma) = \exp [U_1(\bar{q}, \underline{q}, \gamma)] = \underline{q}(\bar{q} - \underline{q})^\gamma.$$

Denoting $\gamma = \frac{\beta_1}{\alpha_1}$, we have the following function :

$$W_1(\bar{q}, \underline{q}, \gamma) = \underline{q}(\bar{q} - \underline{q})^{\frac{\beta_1}{\alpha_1}},$$

and we have :

$$W_2(\bar{q}, \underline{q}, \gamma) = (W_1(\bar{q}, \underline{q}))^{\alpha_1} = \underline{q}^{\alpha_1} (\bar{q} - \underline{q})^{\beta_1}.$$

The Constant Elasticity of Substitution (CES) utility function is defined as follows :

$$W(\bar{q}, \underline{q}, \gamma) = [a\underline{q}^\rho + (1 - a)(\bar{q} - \underline{q})^\rho]^{\frac{1}{\rho}},$$

which can be rewritten as :

$$V_2(\bar{q}, \underline{q}) = (W(\bar{q}, \underline{q}), \gamma)^\rho$$

such that we have :

$$V_2(\bar{q}, \underline{q}, \gamma) = a\underline{q}^\rho + (1 - a)(\bar{q} - \underline{q})^\rho.$$

Defining $\gamma = (1 - a)/a$, V_2 can be transformed into U_2 as follows :

$$U_2(\bar{q}, \underline{q}, \gamma) = \underline{q}^\rho + \gamma(\bar{q} - \underline{q})^\rho, \quad \rho \neq 0.$$

1.4.2 The consumer's demand for goods and for displayed goods

1. In order for the log utility function and the CES function to satisfy Assumption 15, we assume that following conditions hold :

- $\underline{q}_1 < \bar{q}_1/(1 + \gamma)$,
- $\underline{q}_2 > \bar{q}_2/(1 + \gamma^{\frac{1}{\rho-1}})$.

By using the corner solution (1.1), the log-utility and the CES utility functions give respectively the following optimal demands for \underline{q}_i , $i = 1, 2$

$$\underline{q}_i^* = \frac{\Omega}{p}, \quad i = 1, 2$$

and by using the result (1.3) for \bar{q}_i , $i = 1, 2$, we have the following demands for goods \bar{q} for the log and CES functions respectively :

$$\bar{q}_1^* = \frac{\Omega}{p} + \exp \left[\frac{1}{\gamma} \left(v_0(\gamma) - \ln \left(\frac{\Omega}{p} \right) \right) \right], \quad (1.28)$$

$$\bar{q}_2^* = \frac{\Omega}{p} + \left[\frac{1}{\gamma} \left[v_0(\gamma) - \left(\frac{\Omega}{p} \right)^\rho \right] \right]^{\frac{1}{\rho}}. \quad (1.29)$$

Importantly, it is a nice result that in the case of a log utility function, the demand for stock cannot be cleared at all since :

$$\bar{q} - \underline{q} = \exp \left[\frac{1}{\gamma} \left(v_0(\gamma) - \ln \left(\frac{\Omega}{p} \right) \right) \right] > 0.$$

However, in the CES case, there exists a price that clears the stock, which is far from being optimal. This price is given by :

$$p = \Omega / \exp [v_0(\gamma)]$$

2. In order for the log utility function and the CES function to satisfy Assumption 6, we assume that following conditions hold :

- $\bar{q}_1 < \underline{q}_1(1 + \sqrt{\gamma})$,
- $0 < \rho < 1$ and $\bar{q}_2 < \underline{q}_2(1 + \gamma^{\frac{1}{\rho-1}})$.

Regarding the interior solution and applying (1.5), the consumer's demand for \underline{q}_i $i = 1, 2$ is given by :

$$\underline{q}_1^{**} = \frac{1}{1 + \gamma} \bar{q},$$

$$\underline{q}_2^{**} = \frac{1}{1 + (\gamma)^{\frac{1}{1-\rho}}} \bar{q}.$$

Note that in the two previous relations we have respectively :

$$\underline{q}_1^{**} < \bar{q} \text{ and } \underline{q}_2^{**} < \bar{q}.$$

Applying (1.6), the consumer's demand for \bar{q}_i $i = 1, 2$ is given by :¹¹

$$\bar{q}_1^{**} = \frac{1 + \gamma}{\gamma^{\frac{\gamma}{1+\gamma}}} \exp \left[\frac{v_0(\gamma)}{1 + \gamma} \right]$$

$$\bar{q}_2^{**} = \left[\left(\frac{1 + \gamma^{\frac{\rho}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}} \right) v_0(\gamma) \right]^{\frac{1}{\rho}}$$

Both expressions are independent of price. One can easily state the following condition for $\underline{q} \in \left[\alpha, \min \left\{ \frac{\Omega}{p}, \bar{q}_i^{**} \right\} \right]$, $i = 1, 2$:

$$\alpha < \underline{q}_1^{**} < \frac{\Omega}{p} \iff \alpha(1 + \gamma) < \bar{q}_1^{**} < (1 + \gamma) \frac{\Omega}{p}.$$

$$\alpha < \underline{q}_2^{**} < \frac{\Omega}{p} \iff \alpha(1 + \gamma^{\frac{1}{1-\rho}}) < \bar{q}_2^{**} < (1 + \gamma^{\frac{1}{1-\rho}}) \frac{\Omega}{p}.$$

1.4.3 The producer's price

In this subsection, we present the competitive firm. For all that follows, the cost function is defined by

$$TC(\bar{q}) = \frac{1}{2}(\bar{q})^2 - \bar{q}.$$

11. Note that the derivative of the previous expression with respect to γ is of the same sign as $-(v_0(\gamma) + \ln \gamma)$.

1.4.3.1 The competitive price

In the short run, the competitive firm chooses the quantity of displayed goods \bar{q} , for any given price. For the corner solution, the price p_{ic}^* , $i = 1, 2$, is given. The rational competitive firm maximizes its profit with respect to the quantity of displayed goods, which is the solution of the following problem :

$$\max_{\bar{q}} p\bar{q} - \frac{1}{2}\bar{q}^2 + \bar{q}.$$

The first order condition gives :

$$\bar{q}^* = p + 1$$

The rational competitive firm operates a quantity of displayed goods that matches exactly the consumer's solution, and for a log utility function solves :

$$\left[p + 1 - \frac{\Omega}{p} \right] \left[\frac{\Omega}{p} \right]^{\frac{1}{\gamma}} = e^{\frac{v_0(\gamma)}{\gamma}},$$

for which no simple explicit solution is computable.

For a CES utility function, it is not possible to have an explicit general solution. Under the assumption that both the demand for goods q and the quantity of displayed goods \bar{q} are perfect substitutes, that is $\rho \rightarrow 1$, the competitive firm solves :

$$p^2 - \frac{v_0}{\gamma}p - \frac{1 + \gamma}{\gamma}\Omega = 0,$$

such that we obtain

$$p_{2,c}^* = \frac{v_0(\gamma) \pm \sqrt{v_0(\gamma)^2 - 4\gamma(1 + \gamma)\Omega}}{2\gamma}.$$

In the long run, firms enter the market until the zero-profit condition is reached :

$$\bar{q}^* = 2(p + 1)$$

so that the previous prices are divided by 2. Using the interior solution (2.7) for the competitive producer, we obtain the following prices :

$$\bar{q}_{1,c}^s = \frac{2}{1 + \gamma} p_{1,c}^{**},$$

$$\bar{q}_{2,c}^s = \frac{2}{1 + \gamma^{\frac{1}{1-\rho}}} p_{2,c}^{**}.$$

The γ -equilibrium condition is $\bar{q}_1^s = \bar{q}^{**}$, which allows us to determine the competitive market price :

$$p_{1,c}^{**} = 1 + \gamma.$$

$$p_{2,c}^{**} = 1 + \gamma^{\frac{1}{1-\rho}}.$$

1.4.4 The selling-off market

This subsection is devoted to the selling-off market. In this case, the residual consumer is always budget-constrained, since his preference for quantity is zero. The firm has externalized the selling-off activity to another firm, which takes the previous stocks of unsold goods as given. It is important to underline that the selling-off firm can decide not to clear all the stock of unsold goods available at the market period. This decision depends on the optimality of its profit. Hence, the profit of the selling-off firm is given by the following :

$$\pi_{sof} = \tilde{p}_{som} \tilde{q}^s - \frac{1}{2} (\tilde{q}^s)^2.$$

By the free entry condition, the competitive selling-off firm has zero profit (1.19). We have :

$$\tilde{q}_c^{s*} = 2\tilde{p}_c^*.$$

For an equilibrium to exist on the competitive market, the following condition must hold :

$$\tilde{q}_c^* = \tilde{q}_c^{s*} \iff \tilde{p}_c^* = \sqrt{\frac{1}{2}\Omega_r}.$$

1.4.4.1 Curiosities

Below, we present some tables summarizing results for some classical utility functions. We choose these functions because they exhibit some nice theoretical curiosities that we interpret below. Recall that : $r = \bar{q}/\underline{q}$, $TC = 1/2\bar{q}^2 - \bar{q}$ and $\Omega > \Omega_r$.

TABLE 1.1 – Corner and Interior Solutions for the Competitive Market Period

The competitive market period				
Consumer			Competitive firm	
Corner Solutions	\underline{q}^*	\bar{q}^*	\bar{q}_c^*	p_c^*
$U_3 = \ln \underline{q} + \gamma \ln \bar{q}$	$\frac{\Omega}{p}$	$e^{\frac{1}{\gamma}(v_0 - \ln \frac{\Omega}{p})}$	$1 + \sqrt{1 + 2\Omega}$	$\frac{\Omega}{e^{v_0}} [1 + \sqrt{1 + 2\Omega}]^\gamma$
$U_4 = \underline{q}^\rho + \gamma \bar{q}^\rho$	$\frac{\Omega}{p}$	$\left[\frac{1}{\gamma} (v_0 - (\frac{\Omega}{p})^\rho) \right]^{\frac{1}{\rho}}$	$1 + \sqrt{1 + 2\Omega}$	$\frac{\Omega}{[v_0 - \gamma [1 + \sqrt{1 + 2\Omega}]^\rho]^{\frac{1}{\rho}}}$
$U_5 = \underline{q} \bar{q}^\gamma$	$\frac{\Omega}{p}$	$\left[v_0 / (\frac{\Omega}{p}) \right]^{\frac{1}{\gamma}}$	$1 + \sqrt{1 + 2\Omega}$	$\frac{\Omega}{v_0} [1 + \sqrt{1 + 2\Omega}]^\gamma$
Interior Solutions	\underline{q}^{**}	\bar{q}^{**}	\bar{q}_c^{**}	p_c^{**}
$U_6 = \ln \underline{q} + \gamma \ln [\bar{q} - a\underline{q}]$	$\frac{\bar{q}}{a(1+\gamma)}$	$(1 + \gamma) \left[\frac{ae^{v_0}}{\gamma^\gamma} \right]^{\frac{1}{1+\gamma}}$	$\frac{2(a(1+\gamma)+p)}{a(1+\gamma)}$	$r \left[\frac{1+\gamma}{2} \left[\frac{ae^{v_0}}{\gamma^\gamma} \right]^{\frac{1}{1+\gamma}} - 1 \right]$
$U_7 = \underline{q}^\rho + \gamma [\bar{q} - a\underline{q}]^\rho$	$\frac{\bar{q}}{a+(a\gamma)^{\frac{1}{1-\rho}}}$	$\frac{(a+(a\gamma)^{\frac{1}{1-\rho}}) v_0^\rho}{[1+\gamma(a\gamma)^{\frac{1}{1-\rho}}]^{\frac{1}{\rho}}}$	$\frac{2(a+(a\gamma)^{\frac{1}{1-\rho}}+p)}{a+(a\gamma)^{\frac{1}{1-\rho}}}$	$r \frac{v_0^\rho r - 1}{[1+\gamma(a\gamma)^{\frac{1}{1-\rho}}]^{\frac{1}{\rho}}}$
$U_8 = \underline{q} [\bar{q} - a\underline{q}]^\gamma$	$\frac{\bar{q}}{a(1+\gamma)}$	$(1 + \gamma) \left(\frac{av_0}{\gamma^\gamma} \right)^{\frac{1}{1+\gamma}}$	$\frac{2(a(1+\gamma)+p)}{a(1+\gamma)}$	$r \left[\frac{1+\gamma}{2} \left[\frac{ae^{v_0}}{\gamma^\gamma} \right]^{\frac{1}{1+\gamma}} - 1 \right]$

TABLE 1.2 – Results of Stock for the Selling-off Market Period

The competitive selling-off period						
Unsold good $S_c = \bar{q}_c^* - \Omega/p_c^*$	$p(S_c = 0)$	U_r	\bar{q}^*	\tilde{q}_c	\tilde{p}_c	Dead stock
$\frac{(1+\sqrt{1+2\Omega})^{1+\gamma}-e^{v_0}}{(1+\sqrt{1+2\Omega})^\gamma}$	$\frac{\Omega}{1+\sqrt{1+2\Omega}}$	$\ln \underline{q}$	$\frac{\Omega_r}{\tilde{p}_c}$	$2(1 + \tilde{p}_c)$	$\frac{-1+\sqrt{1+2\Omega}}{2}$	$\frac{1+\sqrt{1+2\Omega}^{1+\gamma}-e^{v_0}-(1+\sqrt{1+2\Omega_r})}{(1+\sqrt{1+2\Omega})^\gamma}$
$1 + \sqrt{1 + 2\Omega} - v_0$ $+ \gamma(1 + \sqrt{1 + 2\Omega})^\gamma)^{\frac{1}{\rho}}$	$\frac{\Omega}{1+\sqrt{1+2\Omega}}$	\underline{q}^ρ	$\frac{\Omega_r}{\tilde{p}_c}$	$2(1 + \tilde{p}_c)$	$\frac{-1+\sqrt{1+2\Omega}}{2}$	$\sqrt{1 + 2\Omega} - (\sqrt{1 + 2\Omega_r})$ $-(v_0 - \gamma(1 + \sqrt{1 + 2\Omega})^\gamma)^{\frac{1}{\rho}}$

1.5 Conclusion

The aim of this chapter was to show how stocks of unsold goods can emerge in the economy by changing as little as possible the traditional behavior of both the consumer and the competitive firm through the preference for quantity principle. To do so, we have kept all the usual assumptions relative to the standard microeconomic behavior. We did not introduce uncertainty, imperfect information or asymmetry of information, nor price rigidity, adjustment costs or any type of complexity like wrong expectations or bounded rationality.

Under the preference for quantity principle, given the expressions of the optimal stock of unsold goods, this chapter has provided three main results. This first one has shown that if the main-consumer is budget-constrained, then it may be the case that the competitive firm makes a donation of the stock of unsold goods. The second result implies that if the main-consumer is budget-constrained, then it may be the case that the competitive firm sells the stock of unsold goods at a γ -equilibrium price. Finally, the third one has shown that if the main-consumer is not budget-constrained, the competitive firm has an incentive to destroy (or recycle) the stock of unsold goods.

Since individuals' preference for quantity traps the economy in γ -equilibrium with dead stock, we argue that there is no scope for government intervention during the market period. Nothing can be done against individuals' preferences. However, and more importantly, we argue that there is scope for government intervention on selling-off markets. For example,

the government can provide incentives for organizing the selling-off market efficiently, which in practice is far from being the case. More precisely, since the optimal price prevailing on the selling-off market does not clear the market, and since in certain cases there exists a clearing market price, the government may act as a market clearer.

One can view the first chapter of the thesis as a first attempt to explain the functioning of the selling-off market. In practice, the type of preference for quantity studied here is not the only (or even the most) important cause of unsold goods. We believe it is, however, a significant factor in the observed level of unsold goods and dead stock. Deterministic demand for displayed goods may help to explain supply side frictions. Considering random phenomena as generated by a superposition of deterministic events, we have isolated one of those. In any case, we have shown that neither random demand, adjustment costs, lack of rationality, wrong expectations or imperfect information (or any other similar types of assumption) are necessary to describe, explain and consequently forecast the emergence of unsold goods and dead stock. Finally, to sum up, excess supply (rather than just supply) creates demand.

2 Monopoly and Stock of Unsold Goods : A Based-demand Approach

Abstract

In a monopoly market structure, this chapter shows that the introduction of the preference for quantity principle into economics can allow us to extend the neoclassical equilibrium to a frictionless γ -equilibrium. Such a equilibrium is compatible with stocks of unsold goods and contains as a sub-case the monopoly equilibrium. As in the competitive market structure, it reproduces in the long run the observed GDP regularities that total production exceeds sales, total production co-moves with final sales and production may be more volatile than sales. Applied on labor market functioning, it explains unemployment, Beveridge and Phillips curves emergence without the help of fixed wage or uncertainty.

JEL Classification : D11, D21, D41, D42.

Key words : Microeconomic behavior, firm behavior, economic theory of the consumer, monopoly.

2.1 Introduction

This chapter is in line with the preceding one which aimed to show that a γ -equilibrium exists in a competitive market with perfect information, certainty and price flexibility. In that market equilibrium it is admitted that the main-consumer behavior generates a dead stock of goods for which there is no longer any demand. Regarding the benchmark monopoly market, the theoretical model developed in this chapter enables us to extend the traditional neoclassical equilibrium in which stocks are theoretically absent to a γ -equilibrium in which stocks of goods are theoretically present. However, the latter encompasses the traditional neoclassical equilibrium as a sub-case, by explaining the emergence and persistence of dead stock as an equilibrium phenomenon with perfect information, no uncertainty, price flexibility and full rationality of both firms and consumers in a monopoly market structure.

2.1.1 Related literature

Despite the dominant neoclassical paradigm dealing with the existence of competitive general equilibrium (Smith (1776), Walras (1874), Arrow-Debreu (1954), Uzawa (1961)), where the stock of unsold goods vanishes, a huge economic literature proposes various ways to account for unsold goods in economics, particularly certainty and uncertainty about the consumers demand, Shaffer (1991), Mathewson and Winter (1987), Kawasaki, McMillan, and Zimmermann (1983), Rey and Tirole (1986), Denecker *et al.* (1996). In an uncertainty point of view, these works are capable of explaining many empirical observations, including one that is important to economists : Gross Domestic Product (GDP) fluctuations, with Khan and Thomas (2007). These two authors develop an equilibrium business cycle model, where non-convex delivery costs lead firms to follow (S, s) inventory policies, in line with Herbert E. Scarf (1960) and an optimal (s, S) policy minimizing the average stationary cost in an inventory system, in line with Archibald (1981). Their models underline an important observation for our theory : the existence of co-movement between inventory investment and final sales. It is important to note that interest in inventory behavior or stock of goods as a contributor to aggregate volatility goes back at least to Keynes (1936),

and includes notable contributions by Metzler (1941), Abramovitz (1950), Blinder (1981) and Blinder, Lovell and Summers (1981).

Most of research are concerned with monopoly market equilibrium, where the monopolist power impacts her goods, by taking account of the influence of her production plan on the price of her output, Hart (1985), Bonanno (1990) and Mas-Colell, Winston and Green (1995), Tirole (1988). The monopoly market period is acting without entry of consumers, as in Stokey (1981), Bulow (1982), and Gul, Sonnenschein, and Wilson (1986), or with entry of new consumers as mentioned in Sobel (1991), susceptible to influence the period market. Our work is in line with the preceding works, except that we do not take into account the difference between durable-goods and nondurable goods monopoly. However, some research departs as little as possible from earlier inventory model, where production initiated in current period becomes available for sale only in the next period, with the arbitrary delay providing the only motivation for planned storage, Schutte (1984). In this regard, a monopolist produces a quantity of good that is different from the quantity sold at the market clearing price, and holds a beginning inventory stock of goods remaining at the end the current period.

The monopoly firm faces a linear production cost weighted by a constant unit cost of production. In this model, goods become available for distribution at the same period in which the production decision is made and the cost of production is borne. To show that the stock of unsold goods exists in the monopoly market structure, a static monopoly model based on deterministic demand is used in this document. Given the preference for quantity stated by the consumer's behavior, the non-price-taker firm chooses the optimal price that is compatible with the demand for displayed goods during the period market.

2.1.2 Motivation and objective of the paper

The primary objective of this chapter is to investigate the optimal behavior of a monopolist whose the market power depends not only on the quantity of goods related to

the preference for quantity principle defined in chapter one, but on the quantity of goods that the main-consumer is willing to buy on the period market. Contrary to the traditional assumption that the monopolist optimally chooses the price or quantity that maximizes her profit, this chapter exhibits a case where the monopoly solves her maximizing problem with respect to the consumer behavior, via namely the preference for quantity principle. Hence, the stock of unsold goods available at the end of the market period is impacted by both the consumer and monopolist behaviors.

Abstracting from the marketing literature in which displayed goods generate demand, this chapter explains the reason why facing plays such a huge role in sales. For doing this, we introduce into economics the preference for quantity principle, which allows to capture the consumer's valuation in terms of utility of the available quantity of the displayed goods he decides not to buy¹. The introduction of such a principle into the paradigm generates two separate types of consumers demand. One captures the quantity planned to buy (as usual), the other one captures the quantity of displayed good the main consumer needs to see while buying. The latter demand has never been theoretically studied, despite that its existence has been empirically exhibited by numerous studies in marketing. The first kind of demand stands for the level of sales or the quantity of goods that the consumer really buys, while the second one imposes to the monopoly firm a production constraint. Indeed, monopoly firms have to take into account the demand for displayed goods, since it provides the consumer with a set of services that enter his utility function, which determines his demand for goods.

Theory developed in this thesis is in line with the work of Kahn, McConnell, and Perez-Quiros (2002) who considers the possibility that inventories are a source of household utility when studying GDP fluctuations in the USA during the period 1953-2000. Since in our case displayed goods enter the utility function. With a high displayed quantity of goods, consumers can freely decide how much to buy. This avoids them to return at least once more to complete their shopping. With perfect information, consumers buy a given quantity of

1. Recall that we use the concept of Principle, and not the one of Assumption. Indeed, an assumption refers to the act of taking something for granted or something that is taken for granted, whereas a principle is a general and fundamental truth that may be used in deciding conduct or choice, it is a rule of action.

good from the monopoly producer, who knows the consumer's preferences for quantity, so that he produces a larger quantity of goods, which allows to stimulate demand. Due to this new extra demand, the theory developed in this document is capable of defining an inventory-dependent demand rate which generates (or not) a stock of unsold goods (and, some times more dramatically, dead stock). The novelty here is that it comes from consumer preferences.

2.1.3 Methodology

While studying the existence of the stock of unsold goods, the chapter does not intend to abandon all the relevant theoretical basic neoclassical assumptions, in particular certainty, perfect information, price flexibility, absence of adjustment costs or any bounded rationality. Thus, it is possible to explain the existence of ex-post dead stock through a demand-based explanation instead of supply-based argument in the traditional literature. But the known empirical regularities show that production (at the aggregate level) is always greater than sales in the long run. This reinforces the argument in favor of our quest for the possible deterministic causes of stock accumulation. Even if we accept the assumption of random demand, we should observe that sales sometimes exceed production, generating a sustainable excess demand and no stock accumulation.

2.1.4 Results

The theory we develop has several important implications. First, the model generates the traditional monopoly equilibrium without stocks of unsold goods (and consequently, no need for any selling-off activities), and the monopoly γ -equilibrium with stocks of unsold goods that may not clear on a well-organized selling-off market. In this type of equilibrium, displayed goods become a monopolist's strategic variable.

Second, while analyzing the role of stocks of unsold goods on the monopoly market, we define a modified Lerner index of the firm's market power. If there is no stock of unsold goods, the usual Lerner index applies, but in the presence of unsold goods, it perfectly illustrates how firms manipulate prices or displayed goods to attract consumers'

demand. When consumers are budget-constrained, price is the relevant strategic variable, but when they are not, the quantity of displayed good becomes the strategic variable. Combining both types of consumers leads to a two-dimensional strategic behavior of firms manipulating both price and quantity of displayed good to attract both rational buyers and impulse buyers.

Third, our modeling has the advantage of describing, explaining, and forecasting a lot of observable phenomena, and can be extended in order to explain the emergence of the Phillips curve and the Beveridge curve, stagflation, slumpflation, etc. This is important because due to the existence of dead stock (or equivalently of long-term unemployment on labor market), quite a lot of value-added is not realized and consequently, not reinvested for production or inventories.²

The paper is organized as follows : Section 1 is concerned with the consumer behavior. Section 2 deals with the monopolist behavior relative to both the main-consumer and the residual-consumer, where the main results are presented concerning the existence of the γ -monopoly equilibrium. An alternative modeling with two goods is presented in Section 3 while Section 4 is concerned with different applications with usual utility functions : the Log and *CES* functions. In section 5, some useful discussions about the international non-convergence price migration are presented on involuntary unemployment and the Beveridge curve and the Phillips Curve and the monetary policy. In the last section we conclude.

2.2 The Model

The aim of this section is to build a simply model capable of showing the persistence of stock of unsold goods in the monopoly market. By using the preference for quantity principle, we change as little as possible the traditional behavior of both the consumer and the monopolist. All the usual assumptions relative to the standard microeconomic behavior are unchanged. We assume that the demands are certain, there is perfect information and

2. If inventory coincides with production, the ratio S/I can stand for the rate of inventory investment. This is the difference between goods produced (production) and goods sold (sales) during a given period, and can be applied to the whole economy or to a firm.

all prices are flexible, furthermore, we do not introduce adjustment costs or any type of complexity like wrong expectations or bounded rationality.

As seen in the first chapter, we assume that the main-consumer acts according to the preference for quantity principle whereas the residual consumer is not concerned with it. Regarding the main consumer we distinguish the rational consumer (corner solution) from the impulse buyer (interior solution). There is only one good, consequently there is only one market, namely monopoly market. Contrary to the competitive firm, the monopolist is a price-maker. Despite the latter characteristic relative to the market power, the monopolist faces the two previous kinds of main consumer who have the preference for quantity. The main consumer buys a quantity of goods from the monopoly producer. The fact that main consumers want to see displays of goods (the facing in marketing) when shopping, the monopolist produces the quantity of goods by choosing the level of price in order to stimulate demand (no one wants to enter an empty shop). The latter generates at the end of the market period a stock of unsold good. Thus, the production exceeds sales.³

Since a stock of unsold goods is left at the end of the previous market period, the monopolist chooses to delegate external activities to another firm, called hereafter the external selling-off firm. It takes as given the previous stock of unsold goods remaining and tries to sell it on the selling-off market. Acting in a special type of exchange economy, the selling-off firm is expected to clear the market, which is compatible with the traditional neoclassical equilibrium. However, the model will exhibit situations where the selling-off market will not always be cleared.

2.2.1 The Consumer Behavior

In this section we make a shortcut of the consumer's behavior developed in the preceding chapter. We still suppose that there is perfect information and certainty and individual is perfectly rational. The consumer buys a single good from the monopoly producer and price of this single goods is perfectly flexible. We assume that the consumer still has a

3. We recall " **γ -equilibrium**" any situation where the preference for quantity γ is compatible with an optimal non-zero stock of unsold goods at the end of the market period.

preference for quantity. Thus, there is no range effect or variety effect. With these rational assumptions we aim at changing as little as possible the monopoly market functioning.

Given the above, our model attempts to show the existence of a γ -equilibrium in a monopoly context. Specifically the simple model shows that at the end of the market period the preference for quantity γ generates the stock of unsold goods, which is always not equal to zero. Hence, we assume that on the market the consumer buys a quantity $\underline{q} \in \mathbb{R}_+^*$ if and only if there exists a quantity of displayed goods $\bar{q} \in \mathbb{R}_+^*$ such that $\bar{q} \geq \underline{q}$. This is another kind of equilibrium which implies that if there is an insufficient quantity of displayed goods, no consumer will want to buy anything.

Given Assumption 1 (cf. chapt. 1) related to the consumption set $\mathcal{D}(U)_{\bar{q}, \gamma}$, the consumer's income $\Omega \in \mathbb{R}_+^*$ and the price of the good $p \in (0, \infty)$, the rational main consumer's problem \mathcal{P} is stated as follows⁴ :

$$\mathcal{P} : \left\{ \begin{array}{l} \text{Maximize } U(\bar{q}, \underline{q}, \gamma) \\ \text{w.r.t. } \underline{q} \in \mathcal{D}(U)_{\bar{q}, \gamma} \\ \text{s.t. } p\underline{q} \leq \Omega \\ \underline{q} > 0, \bar{q} > 0, \gamma \neq 0 \end{array} \right.$$

Whatever the utility function, two main cases are distinguished : the corner solution where the budget constraint binds and the utility function is irrelevant, and the interior solution where the utility function is maximized and the budget constraint is irrelevant. The first type of consumer is interpreted as capturing the rational buyer's behavior while the second type allows to capture the impulse buyer's behavior.

The corner solution

According to the assumptions 2, 3 and 4, the consumer's maximization problem has a unique solution denoted by :

$$\underline{q} = \frac{\Omega}{p}. \tag{2.1}$$

4. The concept "main-consumer" was defined in the chapter one.

We assume that the consumer is ready to buy the previous quantity of goods if and only if his utility reaches a certain level denoted by $v(\gamma)$. By replacing the demand for \underline{q}^* in the utility function, knowing that the utility level must be equal to $v(\gamma)$, this allows us to determine the demand for displayed goods \bar{q} . Thus, by keeping the same notation we have :

$$\bar{q}^* = h_\epsilon(\Omega, p, v(\gamma)). \quad (2.2)$$

The interior solution

This paragraph deals with the assumptions that there exists a main consumer's interior solution to the problem \mathcal{P} , where the utility function is maximized and the budget constraint is irrelevant. As mentioned in the introduction of the preceding chapter this kind of consumer is interpreted as capturing the impulse buyer's behavior (Kollat and Willett (1967)).

Since the main consumer is not budget constrained, given the assumption 6, the optimal interior solution may be larger than the optimal solution obtained in the budget constrained case. The non-constrained consumer solves the following maximization problem :

$$\begin{cases} \text{Maximize} & U(\bar{q}, \underline{q}, \gamma) \\ \text{w.r.t.} & \underline{q} \in \mathcal{D}(U)_{\bar{q}, \gamma} \end{cases}$$

Given the lemma 2, the non constrained-consumer's problem admits the following solution \underline{q} and \bar{q} such that :

$$\underline{q}^{**} = \underline{q}(\bar{q}, \gamma), \quad (2.3)$$

$$\bar{q}^{**} = \ell(v_0(\gamma)). \quad (2.4)$$

Furthermore, if $\frac{\partial^2 U(\bar{q}, \underline{q}, \gamma)}{\partial \bar{q} \partial \underline{q}} > 0$ then the demand for good $\underline{q}(\bar{q}, \gamma)$ is increasing in the demand for displayed good \bar{q} .

The Residual consumer

As in the previous chapter we suppose that the residual-consumer buys a quantity $\tilde{q} \in \mathbb{R}_+^*$ in the selling-off market. In this case we use the same problem of the residual consumer that allows to solve the following problem :

$$\begin{cases} \text{Maximize} & \tilde{U}(\tilde{q}_c) \\ \text{s.t.} & \tilde{p}\tilde{q} = \Omega_r, \end{cases}$$

where \tilde{p} is the selling-off price, and $\Omega_r < \Omega$ is the income of the residual consumer. Assuming that the utility function \tilde{U} of the residual consumer is increasing, concave and continuously differentiable, since the residual consumer's budget constraint is bounded, the solution of the consumer's problem is given by :

$$\tilde{q}_k^* = \frac{\Omega_r}{\tilde{p}_k}. \quad (2.5)$$

2.2.2 Monopoly

In the competitive model of the previous chapter we have assumed that both the consumers and the producers were price-takers. Neither the firms nor the consumers can influence the market power. Their choices are setup with respect to the prevailing market price. In this perfect competition if the consumers are budget constrained then the demand and the supply of goods are infinitely elastic with to the market price. However, Economists, investigating the economic aspects of the monopolist's legal rights, have found that they can be reduced to the consequences of the power of the monopolist — as compared with a seller in a competitive market — arbitrarily to decide the price of the commodity, leaving it to the consumers to decide how much they will buy at that price, or, alternatively, to decide the quantity he will sell, by so fixing the price so as to induce consumers to purchase just that quantity (see Lerner (1934)).

In this subsection, we extend the conception of the monopolist's behavior in the direction of γ -equilibrium, in order to account for the emergence of stock of unsold goods

as an optimal output of both the consumers and the monopolist's behavior. Depending on the corner or interior solution of the consumer's problem, the strategic variable for the monopolist is either the price or the quantity of displayed goods.

2.2.2.1 The monopoly producer

The monopoly firm generally takes the market demand function as given and chooses the price and quantity of goods that maximize its profit. However, in view of the main consumer's demand for both the displayed goods \bar{q} and the goods \underline{q} he is willing to buy, the best choice for the monopolist is to maximize his profit with respect to the price p . In doing so, he takes as given both the demand for displayed goods \bar{q} and the demand for goods \underline{q} .

1. Using the consumer's corner solution to problem \mathcal{P} and the results (2.1) and (2.2), the monopoly producer solves the following maximization problem :

$$\begin{aligned} & \text{Maximize } p\underline{q}(\Omega, p) - TC(h_\epsilon(\Omega, p, v(\gamma))), \\ & \text{w.r.t. } p \in \mathbb{R}_+^* \end{aligned}$$

Since the total revenue :

$$R(p) = p\underline{q}(\Omega, p),$$

which is the first term of the objective function, is constant and given that the profit-maximizing firm's behavior is equivalent to that of the cost-minimizing firm behavior, the monopolist solves the following minimization problem :

$$\begin{aligned} & \text{Minimize } TC[h_\epsilon(\Omega, p, v_0(\gamma))] \\ & \text{w.r.t. } p \in \mathbb{R}_+^* \end{aligned} ,$$

Assumption 8. We assume that the total cost TC is convex function in p and continuously differentiable.

By Assumption 8 and from the first order condition, we have the following relation :

$$\frac{\partial TC(\Omega, p, v(\gamma))}{\partial p} = 0.$$

Consider the function $\mathcal{T} : \mathbb{R}_+ \times (0, \infty) \times \mathbb{A}_\varepsilon \rightarrow \mathbb{R}_+$ defined by

$$\mathcal{T}(\Omega, p, v(\gamma)) = \frac{\partial TC(\Omega, p, v(\gamma))}{\partial p}.$$

Assumption 9. For all $\underline{p}, \bar{p} \in (0, \infty)$ such that $\underline{p} < \bar{p}$, $\forall p \in]\underline{p}, \bar{p}[$, \mathcal{T} is of class \mathcal{C}^1 and $\frac{\partial \mathcal{T}(\Omega, p, v(\gamma))}{\partial p} > 0$.

Given the above assumption, the function

$$\mathcal{T}(\Omega, \cdot, v(\gamma)) : [\underline{p}, \bar{p}] \mapsto \mathcal{R}_{\mathcal{T}}$$

is a one-to-one function, where $\mathcal{R}_{\mathcal{T}} = \{\mathcal{T}(\Omega, p, v(\gamma)) : p \in [\underline{p}, \bar{p}]\}$ is the range of \mathcal{T} .

Let us define the function $\zeta : (\Omega, v(\gamma)) \mapsto \zeta(\Omega, v(\gamma))$, whose domain of definition is :

$$\mathcal{D}(\zeta) = \{(\Omega, v(\gamma)) : (\Omega, v(\gamma)) \in \mathbb{R}_+ \times \mathbb{A}_\varepsilon\}$$

and the range is denoted by :

$$\mathcal{R}_\zeta = [\underline{p}, \bar{p}], \text{ with } \underline{p}, \bar{p} \in (0, \infty).$$

Thus we have the following property of global uniqueness :

$$\forall (\Omega, v(\gamma)) \in \mathcal{D}(\zeta), \mathcal{T}(\Omega, \zeta(\Omega, v(\gamma)), v(\gamma)) = 0.$$

Given $(\Omega_0, p_0, v_0(\gamma)) \in \mathbb{R}_+ \times (0, \infty) \times \mathbb{A}_\varepsilon$, the first-order condition of the minimization problem of the monopoly firm implies :

$$\mathcal{T}(\Omega_0, p_0, v_0(\gamma)) = 0$$

By Assumption 9, we have :

$$\frac{\partial \mathcal{T}(\Omega_0, p_0, v_0(\gamma))}{\partial p} > 0.$$

Then, by the Implicit Function Theorem, there exists a neighborhood \mathcal{A} of $(\Omega_0, p_0, v_0(\gamma))$, a neighborhood \mathcal{B} of $(\Omega_0, v_0(\gamma))$ and a unique function $\tau : \mathcal{B} \mapsto \mathbb{R}_+$ of class \mathcal{C}^1 such that :

$$\begin{aligned}
& \bullet p_0 = \tau(\Omega_0, v_0(\gamma)), \\
& \bullet \forall p \in \mathcal{B}, \mathcal{T}(\Omega, \tau(p, v(\gamma)), v(\gamma)) = 0, \\
& \bullet \{(\Omega, p, v(\gamma)) \in \mathcal{A} : \mathcal{T}(\Omega, p, v(\gamma)) = 0\} \\
& = \{(\Omega, \tau(\Omega, v(\gamma)), v(\gamma)) : (\Omega, v(\gamma)) \in \mathcal{B}\}.
\end{aligned}$$

This implies the solution to be the optimal price for the monopolist, denoted hereafter by P_M^* , when the consumer is budget-constrained :

$$p_M^* = \zeta(\Omega, v(\gamma)). \quad (2.6)$$

Given both the demands for \underline{q} and \bar{q} , the monopoly firm chooses the price p_M that minimizes its cost function. This price is fixed by the firm is function of the consumer's revenue and the preference for quantity parameter.

2. Given the interior solution of the consumer and the monopoly price P_M , the rational producer maximizes its concave profit function of class C^1 knowing that he has to produce the quantity of displayed goods \bar{q} in order to sell \underline{q} unit(s) of goods.

$$\begin{aligned}
& \text{Maximize } p_M \underline{q} - TC(\bar{q}) \\
& \text{w.r.t. } \bar{q} \in [\underline{q}, \beta]
\end{aligned}$$

Contrary to the case where the consumer is budget-constrained, the monopoly firm chooses the quantity of displayed goods \bar{q} that maximizes its profit with respect to the main consumer's demand for goods \underline{q} . By using the consumer's solution (2.3), the monopolist solves the following program :

$$\begin{aligned}
& \text{Maximize } p_M \underline{q}(\bar{q}, \gamma) - TC(\bar{q}) \\
& \text{w.r.t. } \bar{q} \in [\underline{q}, \beta].
\end{aligned}$$

The first-order optimality condition is sufficient for an optimum of the monopolist maximization problem :

$$p_M \frac{\partial \underline{q}(\bar{q}_M, \gamma)}{\partial \bar{q}_M} = \frac{\partial TC(\bar{q}_M)}{\partial \bar{q}_M},$$

From this condition we can obtain the optimal monopoly supply for \bar{q}^s :

$$\bar{q}_M^{**} = \bar{q}^s(p_M, \gamma).$$

The quantity of goods that the producer is will to produce depends on the monopoly price and the preference for quantity. Thus the γ -equilibrium condition is obtained if the following condition :

$$\bar{q}_M^{**} = \bar{q}^{**} \iff \bar{q}^s(p_M, \gamma) = \ell(v_0(\gamma)).$$

Hence, the monopolist price is given by :

$$p_M^{**} = \vartheta(v_0(\gamma), \gamma). \tag{2.7}$$

When the consumer is not budget constrained the monopoly firm fixes its price with respect to the preference for quantity and the level of the consumer's utility $v_0(\gamma)$. This result changes as little as possible the monopoly market equilibrium since the producer faces a market structure conditioned by the consumer behavior which is based on the quantity of displayed \bar{q} to buy the quantity of goods \underline{q} .

The Modified Lerner's Index

Traditionally the monopoly firm has the exclusive control of a product in the market. From the point of view of this economic paradigm, a monopolist has market power since the quantity of goods that it is willing to to sell in the market is considered as a continuous function of the price it charges. To measure this market power, the economic literature generally uses the Lerner index. Regarding the assumption that the consumer has a preference for quantity principle it is important to emphasize the related Lerner index.

Lemma 5. Regarding the preference for quantity principle, the new Lerner index that allows to measure the market power is given by :

$$\left[\frac{p(\bar{q}) - \tilde{C}_m(\bar{q})}{p(\bar{q})} \right] = \frac{1}{\varepsilon_{\bar{q}/p}} \frac{1}{\varepsilon_{q/\bar{q}}}, \quad (2.8)$$

Proof. By using the inverse supply function $p(\bar{q})$, the monopoly firm's choice can be reduced to that of \bar{q} . The monopolist's profit can be written as :

$$\begin{aligned} & \text{Maximize } p(\bar{q})q(\bar{q}, \gamma) - TC(\bar{q}). \\ & \text{w.r.t. } \bar{q} \in [\underline{q}, \beta] \end{aligned}$$

The first and second order conditions are given respectively by :

$$\frac{dp(\bar{q})}{d\bar{q}} q(\bar{q}, \gamma) + p(\bar{q}) \frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} = C_m(\bar{q})$$

$$2 \frac{dp(\bar{q})}{d\bar{q}} \frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} + \frac{d^2 p(\bar{q})}{d\bar{q}^2} + p(\bar{q}) \frac{\partial^2 q(\bar{q}, \gamma)}{\partial \bar{q}^2} - \frac{d^2 C(\bar{q})}{d\bar{q}^2} \leq 0$$

The left-hand side of the first above expression gives the marginal revenue of any firm operating a displayed quantity of goods \bar{q} . Then we have :

$$R_m(\bar{q}) = p(\bar{q}) \frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} \left[1 + \frac{\frac{dp(\bar{q})}{d\bar{q}} q(\bar{q}, \gamma)}{\frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} p(\bar{q})} \right]$$

By using respectively the elasticity of market demand and the displayed goods-to-sales ratio :

$$\varepsilon_{\bar{q}/p} = - \frac{d\bar{q}}{d\bar{q}} \frac{p}{\bar{q}} \quad \text{and} \quad r = \frac{\bar{q}}{q(\bar{q}, \gamma)},$$

the marginal revenue of the monopoly firm is given by the following expression :

$$R_m(\bar{q}) = p(\bar{q}) \left[\frac{\partial q(\bar{q}, \gamma)}{\partial \bar{q}} - \frac{1}{r \varepsilon_{\bar{q}/p}} \right]$$

Since the marginal cost is non-negative, we have the following inequality :

$$\varepsilon_{\bar{q}/p} \geq \frac{1}{r \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}}}.$$

Furthermore, since $R_m(\bar{q}) = C_m(\bar{q})$, we have the following expression :

$$r \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}} \left[\frac{p(\bar{q}) - C_m(\bar{q}) / \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}}}{p(\bar{q})} \right] = \frac{1}{\varepsilon_{\bar{q}/p}}.$$

The latter expression can be rewritten as the new index of the degree of monopoly power, which is given by the following expression :

$$\left[\frac{p(\bar{q}) - \tilde{C}_m(\bar{q})}{p(\bar{q})} \right] = \frac{1}{\varepsilon_{\bar{q}/p}} \frac{1}{\varepsilon_{\underline{q}/\bar{q}}},$$

where $\varepsilon_{\underline{q}/\bar{q}} = \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}} \frac{\bar{q}}{\underline{q}(\bar{q}, \gamma)}$ and $\tilde{C}_m(\bar{q}) = \frac{C_m(\bar{q})}{\frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}}}$.

□

The modified Lerner index (2.8) has the following properties. It encompasses the traditional case, when

$$r = \frac{\partial \underline{q}(\bar{q}, \gamma)}{\partial \bar{q}} = 1.$$

In this case it leads to a unit elasticity $\varepsilon_{\underline{q}/\bar{q}} = 1$ so that we obtain

$$\frac{p(\bar{q}) - C_m(\bar{q})}{p(\bar{q})} = \frac{1}{\varepsilon_{\bar{q}/p}},$$

which is the usual expression of the Lerner index obtained in the traditional firm's equilibrium.

The market power of a monopoly in the presence of a preference for quantity is modified as follows. The traditional monopoly uses price as a strategic variable, whereas here price is no longer relevant to a consumer's interior solution. For that reason, the only relevant

strategic variable at the firm's disposal is the quantity of displayed goods. With the help of this strategic variable, the monopoly can attract consumers. We can see this on the right-hand side of the expression (2.8), where the elasticity of the demand with respect to price is now weighted by the elasticity of the demand with respect to displayed goods. Furthermore, the marginal cost is now divided by the sensitivity of the demand with respect to the quantity of displayed goods. Not only does the inverse of the elasticity of demand with respect to price plays a role in the market power, but so does the volatility of demand with respect to the quantity of displayed goods.

It has often been argued that if measured market power deviates from the theoretical Lerner index, it is probably because the monopolist is not acting on purely business principles. If it also has administrative, social, philanthropic or conventional motives, then it may sell commodities below this price if they are considered socially desirable (e.g. public transport authorities), or above this price if they are socially harmful (e.g. State liquor monopolies). Depending on the volatility of demand with respect to displayed goods and on the value of the corresponding elasticity, our market power index may be lower or greater than the Lerner index, so that it helps to reconcile the theory with facts, with the advantage of not abandoning the neoclassical paradigm.

2.2.2.2 The monopoly γ -equilibrium

We now compute the resulting optimal monopoly quantity of unsold goods denoted by S_M .

Theorem 3. Given that consumers have a preference for quantity, under monopoly with perfect information, price flexibility, no uncertainty and full rationality, we obtain the following results :

1. As long as the the consumer is budget-constrained and has a non-negative γ , then there exists a non-zero stock of unsold goods compatible with the optimal monopoly price, or equivalently, there exists (or not) a non-optimal price that clears the stock of unsold goods.

2. If the consumer is not budget-constrained, then there exists a non-zero stock of unsold goods, which is totally independent of any price, and consequently there is no price system that clears the stock of unsold goods.

Proof. Given the corner solution and the N identical budget-constrained consumers, we have :

$$S^* = \sum_{i=1}^N \bar{q}_i^* - \sum_{i=1}^N \underline{q}_i^*.$$

Using the demands for goods given in (2.1) and (2.2), we obtain the following expression of the stock of unsold goods :

$$S^* = \sum_{i=1}^N h_\epsilon(\Omega_i, p, v(\gamma)) - \sum_{i=1}^N \frac{\Omega_i}{p} = NS(\Omega_i, p, v(\gamma)). \quad (2.9)$$

From the interior solution we have :

$$S^{**} = \sum_{i=1}^N \bar{q}^{**} - \sum_{i=1}^N \underline{q}_i^{**}.$$

That implies :

$$S^{**} = N [\bar{q}^{**} - \underline{q}(\bar{q}^{**}, \gamma)].$$

Using the result (2.3), we obtain the following stock of unsold goods :

$$S^{**} = N [\ell(v_0(\gamma)) - \underline{q}(\ell(v_0(\gamma)), \gamma) = NS(v_0(\gamma))]. \quad (2.10)$$

The stock of unsold goods is such that $S(v_0(\gamma)) \neq 0$ and independent of prices. □

As mentioned in the previous chapter on the competitive market, As long as the consumer has a preference for quantity, given the expressions for the optimal stock of unsold goods, we distinguish three main cases.

1. If the consumer is budget-constrained, then the firm may make a donation of the stock of unsold goods.

2. If the consumer is budget-constrained, then the firm may sell the stock of unsold goods at a γ -equilibrium price.
3. If the consumer is not budget-constrained, the firm has an incentive to destroy (or recycle) the stock of unsold goods.

We now present a second model which helps to explain the functioning of the monopolist selling-off market.

2.2.3 The external monopolist selling-off firm

In this subsection, the monopolist studied above is left with the previous stock of unsold goods. As in the case of the competitive market functioning, we assume that the monopolist chooses to delegate the selling-off activity to an external selling-off monopolist. We keep the same set of assumptions as above for competition in the selling-off market, except that the residual consumer has no preference for quantity and is therefore always budget-constrained.

Proposition 3. If the residual consumer has no preference for quantity, then no equilibrium exists. Two main cases are possible :

1. The demand of the residual consumer is lower than the stock of unsold goods S^k and the market still has a stock of unsold goods, called dead stock.
2. The demand of the residual consumer is greater or equal to the stock of unsold goods S^k , and the market clears.

Proof. Recall that on the selling-off monopoly market, the residual consumer's demand is $\tilde{q}_M \in \mathbb{R}_+^*$, since he has no preference for quantity. The market clears if and only if the following condition is satisfied :

$$\tilde{q}_M^*(\Omega, p_M^*) = S^k.$$

This condition specifies the new price that clears the market. This new price is not necessary compatible with the one that the rational selling-off firm optimally chooses. The latter

incurs a storage cost, denoted by $SC(\cdot)$. Thus the profit function of the selling-off firm is π_{sof} :

$$\pi_{sof} = \tilde{p}_M \tilde{q}^s - SC(\tilde{q}^s),$$

where \tilde{q}^s is the quantity of displayed goods.

We now analyze the behavior of the selling-off monopolist. The profit maximization condition implies :

$$p_M = \frac{\partial SC(\tilde{q}^s)}{\partial \tilde{q}^s} \iff \tilde{q}_M^s = q_M^s(p_M). \quad (2.11)$$

The price equals the marginal storage cost. For an equilibrium to exist, it must be the case that (2.5)=(2.11). This leads to the following monopolist market price :

$$\exists \tilde{p}_M^* \in \mathbb{R}_+ \text{ such that } \tilde{q}_M^* = \tilde{q}_M^s \iff \frac{\Omega_r}{\tilde{p}_M^*} = q_M^s(\tilde{p}_M^*),$$

so that we have that the price as a function of the residual consumer's revenue :

$$\tilde{p}_M^* = p_M(\Omega_r).$$

There is no reason for the market clearing condition to hold, and we have :

$$S^k - \tilde{q}^*(\tilde{p}_M^*) \neq 0.$$

Exactly as for the competitive firm, a residual stock of unsold goods remains on the monopoly market. □

2.3 Alternative modeling

This section aims to extend the previous simple based-consumer approach in two directions. First, we consider that the consumer can either be a main consumer on one good and a residual consumer on the other good. Furthermore, we still assume that preference for quantity may play a role for some products, say ${}_2q$ but not for others. Second, we allow the consumer to maximize his surplus.

2.3.0.1 The case of two goods

We now extend our previous simple approach to the case of two goods. Let us consider the case where the consumer consumes $(x, \underline{q}) \in I^2 \subset \mathbb{R}_+^2$. The assumption now is that the rational consumer has no preference for quantity on x , but has γ on \underline{q} . Assuming quasi-linear preferences, we solve :

$$\mathcal{P}_2 \left\{ \begin{array}{l} \text{maximize} \quad x + u(\bar{q}, \underline{q}, \gamma), \\ \text{w.r.t} \quad (x, \underline{q}) \in I \times \mathcal{D}(U)_{\bar{q}, \gamma}, \\ \text{s.t.} \quad x + p\underline{q} \leq \Omega, \\ \quad \underline{q} \leq \bar{q}, \\ \quad \underline{q} > 0, \bar{q} > 0, x > 0. \end{array} \right.$$

Proposition 4.

1. Let us suppose that the utility function defined by :

$$U : I \times \mathcal{D}(U)_{\bar{q}, \gamma} \rightarrow \mathbb{R}, (x, \bar{q}, \underline{q}, \gamma) \mapsto U(x, \bar{q}, \underline{q}, \gamma)$$

and the constraint functions are of class \mathcal{C}^1 . Assuming a regular point (x^*, \underline{q}^*) satisfying the KKT conditions, the problem \mathcal{P}_2 has a solution :

$$\underline{q} = g_\ell(\bar{q}, p, \gamma) \tag{2.12}$$

where p is the monopoly market price.

2. The monopoly firm chooses \bar{q} so as to maximize its profit function

$$\bar{q}_M = h_M(p, \gamma) \tag{2.13}$$

Consequently, assuming quasi-linear preferences and perfect monopoly, there exists a γ -equilibrium for which (x, \underline{q}) is a non-zero solution of problem \mathcal{P}_2 such that $\underline{q} < \bar{q}$.

Proof. The Consumer's Problem

Suppose that (x, \underline{q}) is a regular solution of the problem \mathcal{P}_2 . The \mathcal{L} agrangian of the problem \mathcal{P}_2 is :

$$\mathcal{L}(x, \bar{q}, \underline{q}, \lambda, \mu) = x + u(\bar{q}, \underline{q}, \gamma) + \lambda [\Omega - x - p\underline{q}] + \mu [\bar{q} - \underline{q}].$$

The Karush-Kuhn-Tucker (KKT) conditions are given by the following expressions :

$$\lambda \geq 0, \quad \mu \geq 0,$$

$$\Omega - x + p\underline{q} \geq 0,$$

$$\bar{q} - \underline{q} \geq 0,$$

$$\lambda(\Omega - x + p\underline{q}) = 0,$$

$$\mu(\underline{q} - \bar{q}) = 0,$$

$$1 - \lambda = 0,$$

$$\frac{\partial u(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} - \lambda p - \mu = 0.$$

Let us check for all possibilities of active constraints. Since $\lambda = 1$, we have

$$x + p\underline{q} = \Omega.$$

We necessarily have two cases.

If $\mu = 0$, then the constraint is not active, that is : $\bar{q} - \underline{q} \geq 0$. In this case, we have :

$$\frac{\partial u(\bar{q}, \underline{q}, \gamma)}{\partial \underline{q}} = p. \tag{2.14}$$

The price is equal to the marginal utility with respect to \underline{q} . By using the Implicit Function Theorem, the above equation allows us to determine the following demand functions of the consumer :

$$\underline{q}^* = q_\ell(\bar{q}, p, \gamma), \quad (2.15)$$

$$x^* = \Omega - pq_\ell(\bar{q}, p, \gamma). \quad (2.16)$$

Since the constraint $\bar{q} - \underline{q} \geq 0$ is not active, the consumer's demand satisfies the following condition : $q_\ell(\bar{q}, p, \gamma) < \bar{q}$.

If $\mu \neq 0$ and $\underline{q} = \bar{q}$, then the constraint is active. In order for $\underline{q} = \bar{q}$ and $x = \Omega - p\bar{q}$ to be a feasible solution of \mathcal{P}_2 , we must have the following condition :

$$\frac{\partial u(\bar{q}, \bar{q}, \gamma)}{\partial \underline{q}} \geq p. \quad (2.17)$$

The Monopolist

The problem of the monopolist is as follows :

$$\mathcal{P}_M \left\{ \begin{array}{l} \text{Maximize } pq_\ell(\bar{q}, p, \gamma) - TC(\bar{q}) \\ \text{w.r.t. } \bar{q} \in \mathcal{D}(U)_{.,\underline{q},\gamma} \\ \text{s.t. } q_\ell(\bar{q}, p, \gamma) \leq \bar{q}, \\ \bar{q} > 0, \underline{q} > 0. \end{array} \right.$$

The KKT conditions are :

$$\mu \geq 0,$$

$$\bar{q} - q_\ell(\bar{q}, p, \gamma) \geq 0,$$

$$\mu(\bar{q} - q_\ell(\bar{q}, p, \gamma)) = 0,$$

$$p \frac{\partial q_\ell(\bar{q}, p, \gamma)}{\partial \bar{q}} - TC'(\bar{q}) + \mu(1 - \frac{\partial q_\ell(\bar{q}, p, \gamma)}{\partial \bar{q}}) = 0.$$

If the constraint is not active then we have the following inequality :

$$q_\ell(\bar{q}, p, \gamma) < \bar{q},$$

such that :

$$p \frac{\partial q_\ell(\bar{q}, p, \gamma)}{\partial \bar{q}} = TC'(\bar{q}).$$

By using the Implicit Function Theorem, we obtain the following quantity of good produced by the monopolist :

$$\bar{q}_M = f_\ell(p, \gamma). \tag{2.18}$$

Given the consumer's demand, the quantity of goods depends on the monopoly price and the preference for quantity γ . This quantity satisfies the condition that $q_\ell(\bar{q}, p, \gamma) < \bar{q}$.

If the constraint is active, the following condition must be satisfied to guarantee a feasible solution :

$$\left(\frac{\partial q_\ell(\bar{q}, p, \gamma)}{\partial \bar{q}} - 1\right)^{-1} \left[p \frac{\partial q_\ell(\bar{q}, p, \gamma)}{\partial \bar{q}} - TC'(\bar{q}) \right] \geq 0.$$

In that case, the producer chooses the solution $\bar{q} = q_\ell(\bar{q}, p, \gamma)$, which is beyond the scope of this paper and more a subject of traditional microeconomic textbooks. \square

2.4 Applications with usual utility functions

For an explicit explanation of the monopoly market structure in the presence of the preference for quantity principle we provide not only the consumer's demands and the producer's prices by using the usual utility function examples, but also the optimal stocks. Different cases will be discussed hereafter with respect to both the corner solution and the interior solution for the consumer's maximization problem. Index 1 stands for examples of

the log-utility function, while index 2 captures those of the CES-utility function. As will be show hereafter these two functions exhibit very interesting results.

2.4.1 Preferences

In this subsection, individuals' preferences are captured with a separable utility function : the log-utility function and the CES utility function. Let us first consider the log-utility function.

$$U_1(\bar{q}, \underline{q}, \gamma) = \ln \underline{q} + \gamma \ln(\bar{q} - \underline{q}).$$

Since preferences are invariant with respect to any monotonic transformations of utility, the previous function also encompasses the traditional Cobb-Douglas utility function :

$$W_1(\bar{q}, \underline{q}, \gamma) = \exp [U_1(\bar{q}, \underline{q}, \gamma)] = \underline{q}(\bar{q} - \underline{q})^\gamma.$$

Defining $\gamma = \frac{\beta_1}{\alpha_1}$, we have

$$W_1(\bar{q}, \underline{q}, \gamma) = \underline{q}(\bar{q} - \underline{q})^{\frac{\beta_1}{\alpha_1}},$$

and we have :

$$W_2(\bar{q}, \underline{q}, \gamma) = W_1(\bar{q}, \underline{q})^{\alpha_1} = \underline{q}^{\alpha_1}(\bar{q} - \underline{q})^{\beta_1}.$$

The CES utility function is defined as :

$$W(\bar{q}, \underline{q}, \gamma) = [a\underline{q}^\rho + (1 - a)(\bar{q} - \underline{q})^\rho]^{\frac{1}{\rho}},$$

which can be rewritten as $V_2(\bar{q}, \underline{q}) = (W(\bar{q}, \underline{q}), \gamma)^\rho$:

$$V_2(\bar{q}, \underline{q}, \gamma) = a\underline{q}^\rho + (1 - a)(\bar{q} - \underline{q})^\rho.$$

Defining $\gamma = (1 - a)/a$, V_2 can be transformed into U_2 as follows :

$$U_2(\bar{q}, \underline{q}, \gamma) = \underline{q}^\rho + \gamma(\bar{q} - \underline{q})^\rho, \quad \rho \neq 0.$$

2.4.2 The consumer's demand for goods and for displayed goods

1. In order for the log utility function and the CES function to satisfy Assumption 15, we assume that following conditions hold :

- $\underline{q}_1 < \bar{q}_1/(1 + \gamma)$,
- $\underline{q}_2 > \bar{q}_2/(1 + \gamma^{\frac{1}{\rho-1}})$.

By using the corner solution (1.1), the log-utility and the CES utility functions give respectively the following optimal demands for \underline{q}_i , $i = 1, 2$

$$\underline{q}_i^* = \frac{\Omega}{p}, \quad i = 1, 2$$

and using (2.3) for \bar{q}_i , $i = 1, 2$, we have :

$$\bar{q}_1^* = \frac{\Omega}{p} + \exp \left[\frac{1}{\gamma} \left(v_0(\gamma) - \ln \left(\frac{\Omega}{p} \right) \right) \right], \quad (2.19)$$

$$\bar{q}_2^* = \frac{\Omega}{p} + \left[\frac{1}{\gamma} \left[v_0(\gamma) - \left(\frac{\Omega}{p} \right)^\rho \right] \right]^{\frac{1}{\rho}}. \quad (2.20)$$

Importantly, it is a nice result that in the case of a log utility function, the demand for stock cannot be cleared at all since :

$$\bar{q} - \underline{q} = \exp \left[\frac{1}{\gamma} \left(v_0(\gamma) - \ln \left(\frac{\Omega}{p} \right) \right) \right] > 0.$$

However, in the CES case, there exists a price that clears the stock, which is far from being optimal. This price is given by :

$$p = \Omega / \exp [v_0(\gamma)]$$

2. In order for the log utility function and the CES function to satisfy Assumption 6, we assume that following conditions hold :

- $\bar{q}_1 < \underline{q}_1(1 + \sqrt{\gamma})$,
- $0 < \rho < 1$ and $\bar{q}_2 < \underline{q}_2(1 + \gamma^{\frac{1}{\rho-1}})$.

Regarding the interior solution and applying (1.5), the consumer's demand for \underline{q}_i , $i = 1, 2$ is given by :

$$\underline{q}_1^{**} = \frac{1}{1 + \gamma} \bar{q},$$

$$\underline{q}_2^{**} = \frac{1}{1 + (\gamma)^{\frac{1}{1-\rho}}} \bar{q}.$$

Note that in the two previous relations we have respectively : $\underline{q}_1^{**} < \bar{q}$ and $\underline{q}_2^{**} < \bar{q}$.

Applying (1.6), the consumer's demand for \bar{q}_i $i = 1, 2$ is given by :⁵

$$\bar{q}_1^{**} = \frac{1 + \gamma}{\gamma^{\frac{\gamma}{1+\gamma}}} \exp \left[\frac{v_0(\gamma)}{1 + \gamma} \right]$$

$$\bar{q}_2^{**} = \left[\left(\frac{1 + \gamma^{\frac{\rho}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}} \right) v_0(\gamma) \right]^{\frac{1}{\rho}}$$

Both expressions are independent of price. One can easily state the following condition for $\underline{q} \in \left[\alpha, \min \left\{ \frac{\Omega}{p}, \bar{q}_i^{**} \right\} \right]$, $i = 1, 2$:

$$\alpha < \underline{q}_1^{**} < \frac{\Omega}{p} \iff \alpha(1 + \gamma) < \bar{q}_1^{**} < (1 + \gamma) \frac{\Omega}{p}.$$

$$\alpha < \underline{q}_2^{**} < \frac{\Omega}{p} \iff \alpha(1 + \gamma^{\frac{1}{1-\rho}}) < \bar{q}_2^{**} < (1 + \gamma^{\frac{1}{1-\rho}}) \frac{\Omega}{p}.$$

2.4.3 The producer's price

In this subsection, we present both the competitive firm and the monopoly firm. For all that follows, the cost function is defined by $TC(\bar{q}) = \frac{1}{2}(\bar{q})^2 - \bar{q}$.

2.4.3.1 The monopoly price

For the case of the budget-constrained consumer's result (2.6), the monopoly prices are given by :

$$p_{1,M}^* = \gamma^{\frac{\gamma}{1+\gamma}} \exp \left[-\frac{v_0(\gamma)}{\gamma + 1} \right] \Omega,$$

and

$$p_{2,M}^* = \left[\frac{v_0(\gamma)}{1 + \gamma^{\frac{2\rho}{1-\rho}}} \right]^{\frac{1-\rho}{\rho}} \Omega.$$

5. Note that the derivative of the previous expression with respect to γ is of the same sign as $-(v_0(\gamma) + \ln \gamma)$.

In both the log and CES utility functions examples, the monopoly prices are increasing in the revenue's consumer. When the consumer is budget-constrained the monopoly price is higher for increasing value of the preference for quantity parameter in the interval $]0, \bar{\gamma}]$ for the log-utility function while decreasing when the parameter of preference for quantity increases for the CES-utility function.

For the case of the consumer's interior solution (2.7), the monopoly prices are given by :

$$p_{1,M}^{**} = \frac{(1 + \gamma)^2}{\gamma^{\frac{1}{1+\gamma}}} \exp \left[\frac{v_0(\gamma)}{1 + \gamma} \right]$$

and

$$p_{2,M}^{**} = \left(1 + \gamma^{\frac{1}{1-\rho}} \right) \left[\left(\frac{1 + \gamma^{\frac{\rho}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}} \right) v_0(\gamma) \right]^{\frac{1}{\rho}} .$$

Figures 2.1 and ?? depict evolution of monopoly market price. The first one shows that the market price is increasing in γ when the level of preference v_0 while in the second one the monopoly market price is increasing in both the level of preference for quantity and the level of consumer's preference. Note that these figures are true for higher level of v_0 of indirect utility of the constrained consumer. However, Figures 2.3 and 2.4 show that the result is particularly true for lower level of utility v_0 , for which the monopoly price is convex and strictly increasing.

FIGURE 2.1 – Increase of Monopoly Price Relative to γ

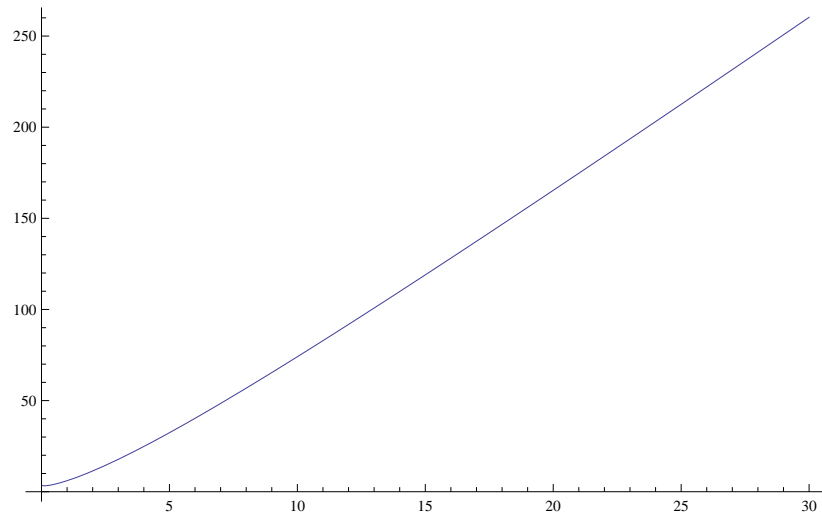


FIGURE 2.2 – Increase of Monopoly Price Relative to γ and v_0

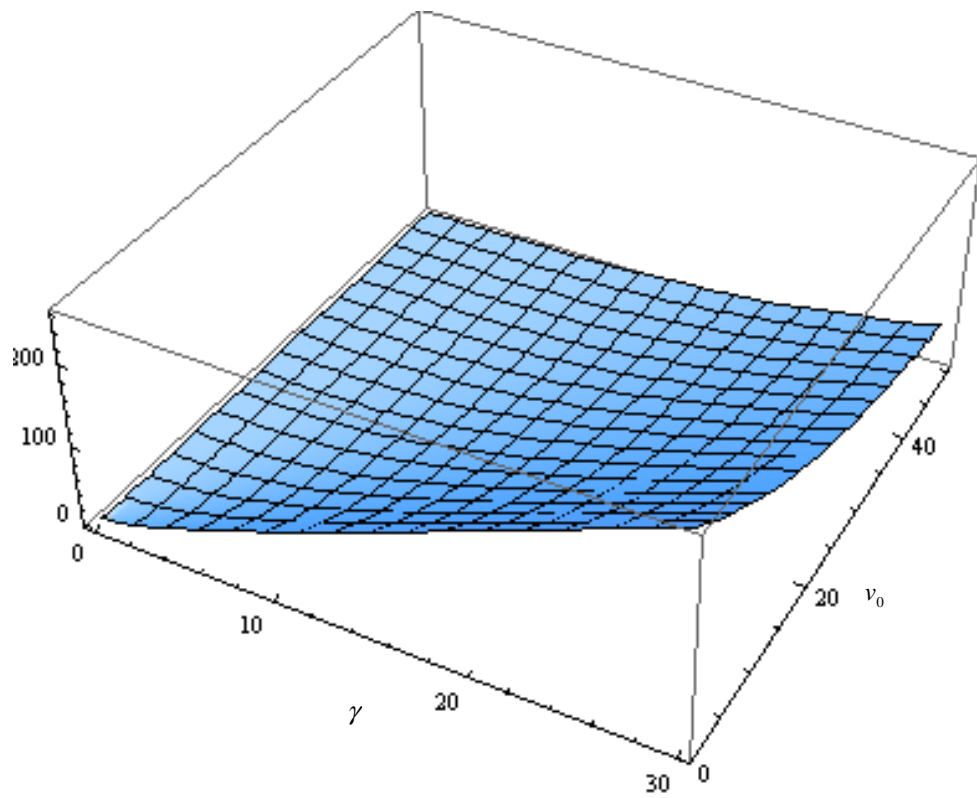
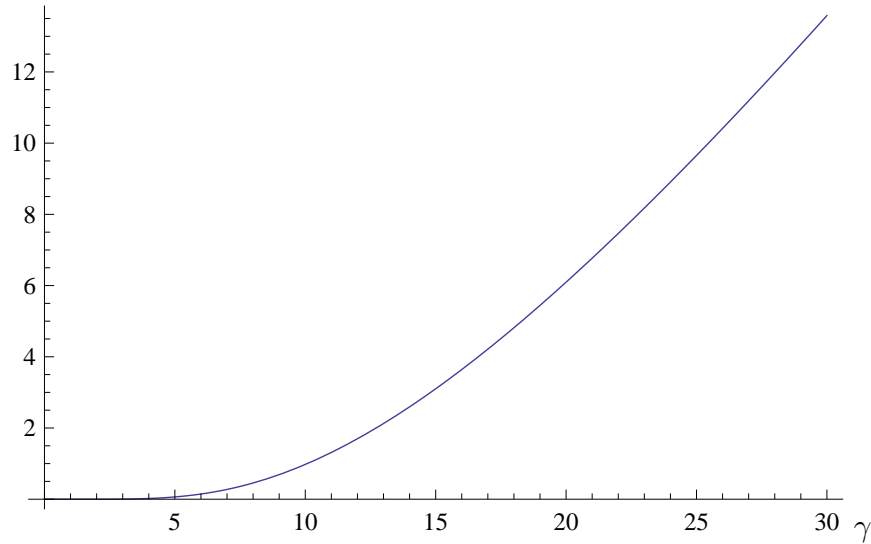


FIGURE 2.3 – Increase of Monopoly Price Relative to γ and lower constant Value of v_0



2.4.4 The optimal stocks

1. The corner solution (2.9) provides us with the following stock of unsold goods :

$$S_1^* = \exp \left[\frac{v_0(\gamma)}{\gamma} \right] \left(\frac{p}{\Omega} \right)^{\frac{1}{\gamma}},$$

It is possible to clear S_1^* by setting $p_{csm} = 0$.

$$S_2^* = \left[\frac{1}{\gamma} \left(v_0(\gamma) - \left(\frac{\Omega}{p} \right)^\rho \right) \right]^{\frac{1}{\rho}}.$$

There exists a selling-off market price.

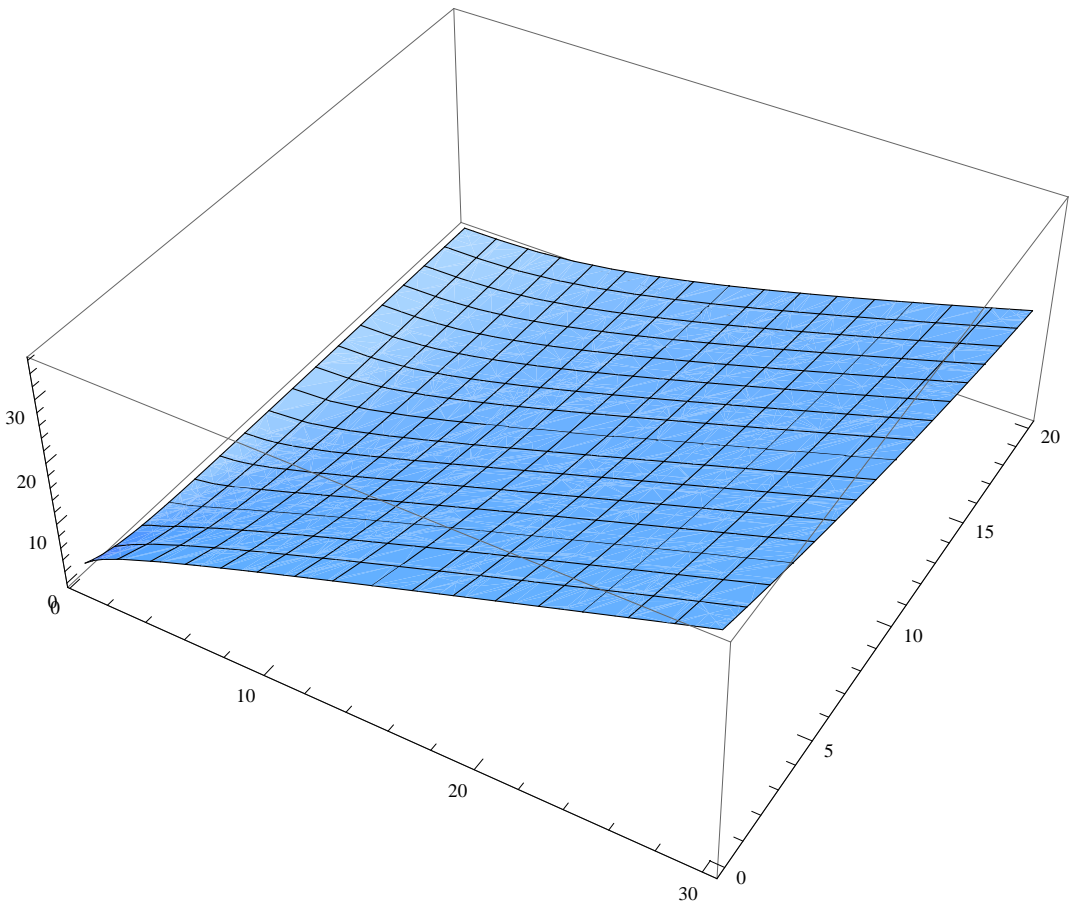
2. The interior solution (2.10) provides us with the following stocks of unsold goods :

$$S_1^{**} = \gamma^{\frac{1}{1+\gamma}} \exp \left[\frac{v_0(\gamma)}{1+\gamma} \right],$$

but with (2.7) the stock from the competitive firm is :

$$S_1^s = \frac{2\gamma}{1+\gamma}.$$

FIGURE 2.4 – Increase of Monopoly Price Relative to γ and lower Value of v_0



These two stocks must be equal, which gives the level of the utility :

$$v_{1,0}(\gamma) = \gamma \log \gamma - (1 + \gamma) \log(1 + \gamma).$$

Following the same reasoning for the CES utility function gives :

$$S_2^{**} = \frac{\gamma^{\frac{1}{1-\rho}}}{(1 + \gamma^{\frac{1}{1-\rho}})} \left[\left(\frac{1 + \gamma^{\frac{\rho}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}} \right) v_0(\gamma) \right]^{\frac{1}{\rho}}.$$

The stock from the competitive firm's point of view is :

$$S_2^s = \frac{2\gamma^{\frac{1}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}}.$$

The compatible utility level is :

$$v_{2,0}(\gamma) = 2^\rho \left[\frac{1 + \gamma^{\frac{1}{1-\rho}}}{1 + \gamma^{\frac{\rho}{1-\rho}}} \right].$$

There is no selling-off market price. Minimizing the stock with respect to $v_0(\gamma)$ is not a solution. If the preference for quantity is zero, then there is no stock of unsold goods at all.

Clearly there are simple conditions under which these quantities of unsold goods are positive in general.

The following consumer's interior solution illustrates the above results. We have shown in this model that it may be the case that a positive quantity of unsold goods exists in equilibrium and we call such a situation a γ -equilibrium.

2.4.5 The selling-off market

In the selling-off market the residual consumer is always budget-constrained, since his preference for quantity is zero. The monopolist has externalized the selling-off activity to another firm, which takes the previous stocks of unsold goods as given. It is important to underline that the selling-off firm can decide not to clear all the available stock of unsold goods. This decision depends on the optimality of its profit. The profit is :

$$\pi_{sof} = \tilde{p}_{som} \tilde{q}^s - \frac{1}{2} (\tilde{q}^s)^2.$$

The monopolist's profit maximization leads to (2.11). The first order condition gives :

$$\tilde{q}_M^{s*} = \tilde{p}_M^*.$$

For an equilibrium to exist on the market, the following condition must hold :

$$\tilde{q}_M^* = \tilde{q}_M^{s*} \iff \tilde{p}_M^* = \sqrt{\Omega_r}.$$

2.4.5.1 Curiosities

Below, we present some tables summarizing results for some classical utility functions. We choose these functions because they exhibit some nice theoretical curiosities that we interpret below. Recall that $r = \bar{q}/q$, that $TC = 1/2\bar{q}^2 - \bar{q}$ and that $\Omega > \Omega_r$.

TABLE 2.1 – Corner and Interior Solutions for the Monopoly Market Period

The monopoly market period			
Consumer		Monopoly firm	
Corner Solutions	\underline{q}^*	\bar{q}^*	p_M^*
$U_3 = \ln \underline{q} + \gamma \ln \bar{q}$	$\frac{\Omega}{p}$	$e^{\frac{1}{\gamma}(v_0 - \ln \frac{\Omega}{p})}$	$\frac{\Omega}{e^{v_0}}$
$U_4 = \underline{q}^\rho + \gamma \bar{q}^\rho$	$\frac{\Omega}{p}$	$\left[\frac{1}{\gamma} \left(v_0 - \left(\frac{\Omega}{p} \right)^\rho \right) \right]^{\frac{1}{\rho}}$	$\frac{\Omega}{(v_0 - \gamma)^{\frac{1}{1-\rho}}}$
$U_5 = \underline{q} \bar{q}^\gamma$	$\frac{\Omega}{p}$	$\left[v_0 / \left(\frac{\Omega}{p} \right) \right]^{\frac{1}{\gamma}}$	$\frac{\Omega}{v_0}$
Interior Solutions	\underline{q}^{**}	\bar{q}^{**}	p_M^{**}
$U_6 = \ln \underline{q} + \gamma \ln [\bar{q} - a\underline{q}]$	$\frac{\bar{q}}{a(1+\gamma)}$	$\frac{a(1+\gamma)+p}{a(1+\gamma)}$	$r \left[\frac{1+\gamma}{2} \left[\frac{ae^{v_0}}{\gamma^\gamma} \right]^{\frac{1}{1+\gamma}} - 1 \right]$
$U_7 = \underline{q}^\rho + \gamma [\bar{q} - a\underline{q}]^\rho$	$\frac{\bar{q}}{a+(a\gamma)^{\frac{1}{1-\rho}}}$	$\frac{(a+(a\gamma)^{\frac{1}{1-\rho}})v_0^\rho}{\left[1+\gamma(a\gamma)^{\frac{1}{1-\rho}} \right]^{\frac{1}{\rho}}}$	$r \frac{v_0^\rho r - 1}{\left[1+\gamma(a\gamma)^{\frac{1}{1-\rho}} \right]^{\frac{1}{\rho}}}$
$U_8 = \underline{q} [\bar{q} - a\underline{q}]^\gamma$	$\frac{\bar{q}}{a(1+\gamma)}$	$(1+\gamma) \left(\frac{av_0}{\gamma^\gamma} \right)^{\frac{1}{1+\gamma}}$	$r \left[\frac{1+\gamma}{2} \left[\frac{ae^{v_0}}{\gamma^\gamma} \right]^{\frac{1}{1+\gamma}} - 1 \right]$

TABLE 2.2 – Results of Stock of Unsold Goods for the Monopoly Selling-off Period

The monopoly selling off period					
Unsold good $S_M^* = \bar{q}^* - \Omega/p_M^*$	$\tilde{p}^*(S_M^* = 0)$	U_r	\tilde{q}^*	\tilde{p}_M	Dead stock
$1 - e^{v_0}$	$\Omega > \Omega_r$	$\ln \underline{q}$	$\frac{\Omega_r}{\tilde{p}_e}$	$\frac{-1+\sqrt{1+4\Omega}}{2}$	$1 - e^{v_0} - \frac{(1+\sqrt{1+4\Omega})}{2}$
$1 - (v_0 - \gamma)^{\frac{1}{\rho}}$	$\Omega > \Omega_r$	\underline{q}^ρ	$\frac{\Omega_r}{\tilde{p}_e}$	$\frac{-1+\sqrt{1+4\Omega}}{2}$	$1 - (v_0 - \gamma)^{\frac{1}{\rho}} - \frac{(1+\sqrt{1+4\Omega})}{2}$

2.4.5.2 Economic interpretations

Usual models of monopoly do not explicitly deal with budget-constrained consumers. Most of the time, the demand is assumed to be linearly decreasing in price. Here, we have a non-linear demand for goods, associated with a non-linear demand for displayed goods. In the above examples, consumer's corner solutions generate a higher price in competition than in monopoly. One should not be surprised by such a result. This curiosity comes directly from the fact that the optimal quantity of displayed goods is always increasing in price. Consequently, since the monopoly has no supply curve, and because it takes the consumer's demand as given, \bar{q}^* acts as a supply curve. The competitive optimum is such that $\bar{q}_c^* > 1$ so that we have in equilibrium

$$\bar{q}_c^* AC > AC \iff TC > AC.$$

Since, in competition, the minimization of total cost indicates the optimal quantities to produce and display, the free entry condition implies that the price crosses $rAC < TC$, and we obtain the result. Everything happens as if in competition, firms have to display more goods than in monopoly, which is consistent with the view that display goods become a strategic variable. Such behavior associated with increasing returns to scale should have the result of increasing output while decreasing price, and consequently improving the social welfare of consumers facing monopoly. We underline here one of the possible active forces on the market. Note that for $\gamma = 1$, we can easily solve equation (1.28) and prove that the competitive price may be lower than the monopoly price. So the exhibited curiosities are not a general theoretical phenomenon. When the demand for displayed goods is decreasing in price (see Figures 3,6 for instance), traditional conclusions hold. An increase in production generates a decrease in price.

2.5 Discussion

This section extends the potential of our modeling in various economic directions.

2.5.1 International price non-convergence and international migration

Let us consider two countries in autarky. One is populated with consumers having a high preference for quantity, as is probably the case in most of the developed countries. In the other, consumers have a low preference for quantity, which may capture an underdeveloped country. In an open-border world, our modeling helps to explain why the two domestic prices do not converge to some international price level. It is because in the developed country, the preference for quantity generates a higher competitive equilibrium price that cannot be lowered by any means, so that dead stock exists and cannot be eliminated. The less developed country is characterized by a lower competitive equilibrium (with little or no dead stock), due to a low preference for quantity. Even within a competitive framework, price does not fall or rise to converge across countries. That is why an international institution capable of organizing the international selling-off market could play an important role by redistributing dead stock among countries, could play contributing to the international price convergence. Alternatively, if one allows for international migration, then possibly a post-migration competitive equilibrium may emerge in the integrated economy. Such a post-migration equilibrium lies somewhere between the two autarkic equilibria exhibited in our modeling. We do not develop this line of research here, since it is a complete program of research in itself.

2.5.2 Involuntary unemployment and the Beveridge curve

An interesting application of the principle of preference for quantity on labor markets is the following. Following Walras' conception of the manager behaving according to his utility, (see p. 236, §227), let us suppose, exactly as in our modeling, that in order to hire n workers, producers (the demand side of the labor market) prefer to choose among N available workers. In that case, frictionless involuntary unemployment emerges. No economic policy can clear the market, since fighting against individuals' preferences is not an option for the government. Low human capital markets are more competitive than high ones. Whatever the labor market structure is, our modeling suggests that the market period ends up with unemployment. On the selling-off market (the labor market of unemployed

workers), firms accept to hire workers with lower human capital at a lower wage, but unfortunately, some of the workers will experience long-term unemployment. Symmetrically, let us now suppose that the supply side has a preference for quantity. Workers prefer to accept a job when they have been contacted by J identical firms, exactly as in search theory (see Gaumont, Schindler and Wright (2006) for more details). Here again, our modeling generates a frictionless explanation of the non-matched jobs compatible with the Beveridge curve. If we combine both arguments, involuntary unemployment coexists with the Beveridge curve.

2.5.3 Monetary policy and the Phillips curve

In this section, we suppose that γ -equilibrium coexists in labor and in goods markets simultaneously. Let us analyze the labor market functioning under γ -equilibrium, which is not as simple as in the traditional competitive equilibrium, since now unemployment emerges. The demand side is constituted by firms, and the supply side by workers. As in the previous section, suppose that there are N workers, and that preference for quantity principal is at work. The firm wants to hire $n < N$ individuals. More precisely, firms want to "interview" more workers than what they truly need to hire for one vacant job. This is known in labor market to be a screening device among similar applicants⁶. Exactly like in the core of the paper, we know that in γ -equilibrium, there exists a stock of workers called unemployment. Note that endogenous involuntary unemployment is generated without the help of fixed wage or uncertainty.

Let us now interpret the income of consumers as being a certain quantity of money $M = \Omega$, as in cash-in-advance models. As long as money is exogenously created, whatever the market structure is, prices increase in the economy. But it is worth noting that for budget-constrained consumers, as illustrated in Section 4, there are three cases. Suppose that in γ -equilibrium, both the demand for goods and the demand for displayed goods are downward sloping. An exogenous increase in the money supply results in either a smaller price increase or a greater price increase, depending on the value of parameters such as the

6. For hiring one top CEO, firms are screening over 1500 applicants on average.

FIGURE 2.5 – Monetary policy and the Phillips curve

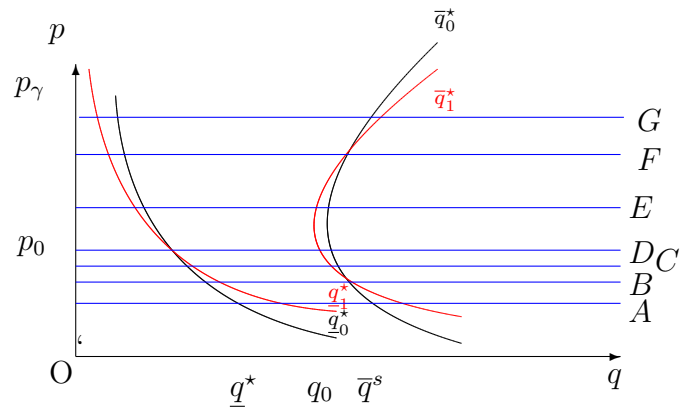


Figure 6 : In red the new demand for goods and for displayed goods after money increase. For corner solution, and $U_1 = \ln$ or $U_2 = CES$ utility functions, see Section 4. level of consumer income.

In the case where one percent money creation engenders a less than one percent price increase, both the demand for goods and production are upward shifting. The reciprocal case (where a one percent wage increase engenders a more than one percent price increase) is left to the reader. The following figure illustrates how curves shift after both effects : the money increase and the price increase. The demand for goods rotates around the value $\underline{q}^* = 1$, while the demand for displayed goods \bar{q}^* is modified as shown in the figure :

For a given price, the increase in money shifts the M/p curve up to

$$(M + dM)/p = M_1/p$$

and the new curve has a stronger slope everywhere. At the new level of money M_1 , the price adjustment provokes a downward move of the curve which becomes $M_1/(p + dp)$ with a new smoother slope everywhere. The final result is that the demand for goods has rotated around the value $q = p = 1$ (the black curve becomes the red one).

Assuming that the labor market is compatible with a γ -equilibrium, the analysis is

broken down into seven main cases as follows :

1. In the first case *A*, we suppose that both curves are decreasing such that the red curves are above the black curves. Consider any γ -equilibrium in this region.

After the change in money supply, and the resulting price adjustment, the new γ -equilibrium is such that it exhibits both an increase in sales and an increase in displayed goods. Sales and production co-move, so that if production is an increasing function of labor, expansion leads to a fall in unemployment, reflecting the so-called Phillips Curve. Price increases are associated with a fall in unemployment. Note that if the increase in money supply is high enough, then consumers may switch from budget-constrained behavior to unconstrained behavior. In that case, the economy turns to stagflation.

2. The second case *B* captures a situation where an increase in the money supply provokes inflation, but the demand for goods shifts upward, while the demand for displayed goods remains about the same :

$$dM > 0 \Rightarrow d\underline{q} > 0$$

in the short run, but in the long run, the price increase generates $d\underline{q} < 0$. By our assumption that $dp < dM$, the final effect is $d\underline{q} > 0$, while $d\bar{q} = 0$. Sales growth does not convert into either production or employment growth, which is also characteristic of stagflation.

3. The third case *C* also has interesting results, when an increase in the money supply with inflation leads to an increase in the demand for goods but a decrease in production, thus increasing unemployment. This is achieved for any p_γ very closed down p_0 . Such a situation is characteristic of slumpflation, where inflation is associated with an increase in unemployment. Sale and production co-move.
4. The fourth case *D* is such that price are fixed, no inflation, sales remain constant, but production is reduced, generating unemployment.

5. The fifth case E exhibits a situation where sale, price and production decrease together. Sale and production co-move. Price reduction is associated with an increase in unemployment.
6. The sixth case F captures a situation where sales decrease, price too, but output remain constant. unemployment is unchanged in the economy.
7. Finally, the seventh case G is such that price and sales are reduced, while output increase. Unemployment is reduced. The Phillips curve is up-ward sloping.

Let us now turn to the case where consumers are not budget-constrained, and there is no clear relation between money increase and price increase. Postulating the existence of the same type of relation as described above, $0 < dp < dM$, there is only one main case. The demand for goods and the demand for displayed goods remain unchanged, generating no sales or output growth but only inflation (see Figure 6). Production costs increase, without any other associated effect on the economy. Sales growth is never converted into output expansion, and as a consequence never results in unemployment reduction, again reflecting stagflation. On the other hand, workers will become more selective about the jobs they accept, shifting the Beveridge curve upwards.

2.6 Conclusion

This chapter has explored the role of the preference for quantity in the emergence of γ -equilibrium on monopoly market structure. We have argued that when displayed goods provide the consumer with some kind of service, in a perfectly flexible world, market equilibrium will be characterized by stocks of unsold goods at the end of the market period. As it was in the chapter one, when extending the competitive market period to the selling-off period, we have shown how stocks of unsold goods may be transformed into dead stock.

A surprising result is that these stocks are optimal, since they are not caused by any type of market failure. In frictionless market structures, we have provided a demand explanation of the following regularities : production is greater than sales, production co-moves

with sales, production is more volatile than sales in the neighborhood of the γ -equilibrium, dead stock emerges on the selling-off market and may end up being donated to charities, destroyed or recycled. We have identified several forces at work, some which tend to increase the displayed-goods-to-sales ratio associated with the monopoly market γ -equilibrium, while others tend to make it smaller.

In our theory, each monopolist manipulates both price and quantity of displayed goods to attract consumers. Since displayed goods are a strategic variable, a natural extension of the Lerner index is proposed, which takes into account the novel idea of a demand for displayed goods related to the demand for goods. As long as these two demands are decreasing in price, our theory extends the traditional one to allow for dead stock in equilibrium. Competition is welfare-improving compared with monopoly. However, if the demand for displayed goods increases in price (and we have shown this is possible), then competitive equilibrium, even if it produces more goods than monopoly, may generate higher prices. Displayed goods appears to act as fixed costs so that there are cases where large firms, by displaying less goods than small ones, can be more efficient, and can then price goods below the competitive γ -equilibrium.

Résumé :

Descripteurs :

Title and Abstract :

Keywords :

Nota : cette page, dernière de couverture, sera retournée avant reliure.

.1 Conclusion

The main objective of this document was to construct a theoretical model which shows how stocks of unsold goods can emerge in the economy by changing the standard microeconomic behavior. However, we have kept all the usual assumptions relative to the standard microeconomic behavior: no uncertainty, no imperfect information or asymmetry of information, nor price rigidity, adjustment costs or any type of complexity like wrong expectations or bounded rationality.

To do so, we introduced into the utility function a parameter of preference for quantity which permits to obtain the expressions of the optimal stock of unsold goods. Indeed, three main results are provided. This first one has shown that if the main-consumer is budget-constrained, then it may be the case that the competitive firm makes a donation of the stock of unsold goods. The second result implies that if the main-consumer is budget-constrained, then it may be the case that the competitive firm sells the stock of unsold goods at a γ -equilibrium price. Finally, the third one has shown that if the main-consumer is not budget-constrained, the competitive firm has an incentive to destroy (or recycle) the stock of unsold goods. In any case, the price that enables the stock of unsold goods to be zero is not optimal for the competitive firm.

Since individuals' preference for quantity traps the economy in γ -equilibrium with dead stock, we argue that there is no scope for government intervention during the market period. Nothing can be done against individuals' preferences. However, and more importantly, we argue that there is scope for government intervention on selling-off markets, in order to provide incentives for efficiently organizing this market. This situation departs from being the case in the established tradition neoclassical economy.

We have also explored the role of preference for quantity in the emergence of γ -equilibrium on monopolistic market structure. We have argued that when displayed goods provide the consumer with some kind of services, in a perfectly flexible world, market equilibrium will be characterized by stocks of unsold goods at the end of the monopolistic

market period. By extending the monopolistic market period to the selling-off period, we have shown how stocks of unsold goods may be transformed into dead stock.

In our theory, each monopolist manipulates both price and quantity of displayed goods to attract consumers' demand. Since displayed goods are a strategic variable, a natural extension of the Lerner index was proposed, which takes into account the novel idea of a demand for displayed goods related to the demand for goods. As long as these two demands are decreasing in price, our theory extends the traditional one to allow for stock of unsold goods in equilibrium.

Furthermore, competition is welfare-improving compared with monopoly. However, if the demand for displayed goods increases in price, even if competitive equilibrium produces more goods than monopoly, may generate higher prices. Displayed goods appears to act as fixed costs so that there are cases where large firms, by displaying less goods than small ones, can be more efficient, and can then price goods below the competitive γ -equilibrium.

Deuxième partie

Econometric Evidence and Overlapping Generations Economy

.2 Introduction

The purpose of this part is to exhibit some applications for the main theoretical results stemmed from the preceding chapters. In this regard, it is divided into two chapters. The first one presents empirical evidence showing that the preference for quantity has a significant impact on the production and stock of unsold goods. Based on the demand-side, the theoretical results obtained in the first part of the thesis are fitted to the quarterly U.S. data in order to test the robustness of our theoretical model. The second chapter investigates a two-country overlapping generations economy with stock of unsold goods and money creation by the government. We analyze the steady-state implications of capital mobility in a context of the principle of preference for quantity.

In addition to theoretical and empirical literature proposing a supply-side explanation of the existence of unsold goods through the assumption of uncertain demand (delay between production and sales), we focus our attention on the existence of preference for quantity as one of explanation of the stock of goods, or largely in the inventory investment. This based-demand assumption allows us to give clearer explanation of GDP fluctuation.

The motivation for such an empirical approach based on the emergence of stock of unsold goods through the preference for quantity allows us to exhibit a deeper explanation of the true cause of economic fluctuations. Contrary to earlier research, our modeling seeks to attribute the volatility of economic fluctuations to change in inventories investment via the demand-side based on the preference for quantity by fitting the theoretical results obtained in Blot, Cayemitte and Gaumont (2011) with the U.S. data via econometric considerations.

To do so, we take into account the specification of the solution of maximization problem, which allows us to capture both the households' behavior and firms' behavior. Since models issued from preceding results match a nonlinear relation with respect to variables and parameters it is impossible to estimate them by using the traditional linear squares (OLS). The methodology used to estimate this kind of models stemmed from the solution of

individuals maximizing problem is the Gauss-Newton regression. The latter helps compute nonlinear least squares (NLLS) estimators of the econometric models.

Otherwise we consider a two-country overlapping generations model based on the principle of preference for quantity and liquidity constraint. Such a principle explains the emergence and persistence of stock of unsold goods as an equilibrium phenomenon with perfect information, no uncertainty, price flexibility and full rationality of both consumers and firms. It allows to extend the dynamic OLG equilibrium to a frictionless dynamic OLG γ -equilibrium – compatible with stocks of unsold goods – that includes as a sub-case the neoclassical equilibria.

In our model, countries 1 and 2 differ from the level of preference for quantity. Assuming that individuals in country 1 have higher level of preference, the last chapter analyzes the domestic stock of unsold goods based on inflation and the excess of money creation. In our model each individual born in country 1 as well as in country 2 has a preference for quantity which deals with mobility of capital across countries. Regarding this preference acted in the first period of lifetime, our modeling departs as little as possible from the traditional paradigm that all markets are in equilibrium according to Walras' Law.

Despite that the economic literature proposes various ways to account for unsold goods in economics, there are no works on overlapping generations models dealing with the stock of unsold goods. The model developed here with money and the stock of unsold goods provides necessary condition for the local stability of the autarkic steady-state γ -equilibrium to exist. Thus if the preference for quantity stated at the first period of individuals lifetime does not hold, then the local steady-state is equivalent to that of the traditional results.

In the context of international capital mobility, the present chapter discusses briefly how consumers are willingness to care about current consumption relatively to future consumption. The consumer's choice depends not only on his preference for quantity but also the anticipated rate of return on savings invested abroad.

In an attempt to understand individuals reactions with respect to cash in advance constraint and the perfect-foresight anticipations on capital invested abroad, our model

shows that the steady-state welfare is increasing in the domestic demand for capital and the capital mobility. Finally, this chapter studies the impact of monetary policy on dynamics of capital and on individuals' behavior in an open OLG economy.

A Preference for quantity : A Based-demand Empirical Approach

This paper presents an empirical evidence showing that the preference for quantity principle has a significant impact on the production and stock of unsold goods. Based on the demand-side, the theoretical results obtained in Blot, Cayemite and Gaumont (2011) are fitted to the quarterly U.S. data and pointed out that the stock of goods has a significant impact on the economic fluctuations.

JEL Classification : C01, C51, D11, E32.

Key words : GDP Fluctuations, Microeconomic behavior, Nonlinear Regression, Gauss-Newton Method.

A.1 Introduction

This paper presents an empirical evidence showing that the preference for quantity principle has a significant impact on the production (or GDP) and stock of unsold goods. It is admitted in the theoretical version that this new concept in economics is acting and thus measurable whatever the markets structure with certainty, perfect information, prices flexibility and without adjustment costs. Recall that this preference for quantity principle allows to capture the consumer's valuation in terms of utility of the available quantity of the displayed good he decides not to buy. Based on the management and marketing literature, the theoretical argument encompasses the neoclassical approach without stock of goods and points out the role of stock of unsold goods in real business cycles (RBC)¹. The concept of *inventory* is used to refer to the stock of goods. Generally, we mean by inventory a list of goods and materials available in stock by businesses².

Most empirical research account for shocks as the main cause of the bulk of fluctuations in inventories or stock of goods, Metzler (1941), Lundberg (1955), Lovell (1961, 1962), Blanchard (1983), Blanchard and Quah (1988) and West (1990). These works have in general emphasized the important role of the shocks related to those of cost and demand, Blinder (1986b), Maccini and Rossana (1984), Miron and Zeldes (1987). Indeed, in his paper, West (1990) has used a simple real linear-quadratic inventory model to determine how cost and demand shocks interacted to cause fluctuations in aggregate inventories and GNP in the United States from 1947 to 1986. Since production is more variable than final sales, the cost shocks are considered as the the more important source of fluctuations in inventories.

Production-smoothing models introduced by Charles Holt, Franco Modigliani, John Muth and Herbert Simon (1960) are also concerned with the role of inventory on the eco-

1. We can find a description of the approach to model evaluation found in much of the RBC literature in Kydland and Prescott (1996) and a critical appraisal of that approach in Christopher A. Sims (1989, 1996)

2. This definition is in line with that USA and Canada. However, the concept is equivalent to *stock* in British English. In the rest of the paper, this term cannot be, in any case, interpreted as an *asset* relatively to inventory or stock in Accounting, since the latter is not treated in this version of the paper.

conomic fluctuations. These four authors suggest that because of increasing marginal costs of production, the desire to smooth production relative to demand will also cause adjustment of inventories in response to demand. However, based on Hay's argument (1970), Darling and Lovell (1971) argue that production smoothing does not represent an acceptable substitute to the flexible accelerator. Both hypotheses are consistent with maximizing behavior, and "the choice between the two alternative approaches must be resolved empirically. They show that serious specification error may result from the use of the flexible accelerator rather than production-smoothing in inventory studies, Ghali (1974).

Due to limitations of the previous papers, generalized-cost functions are used to exhibit some effect of inventory demands on the economic fluctuations, Ramey (1989). The work of Blanchard (1983) – based on an empirical study of the inventories behavior in the automobile industry – shows that the variance of production is larger than that of sales, as theoretically shown in Blot, Cayemitte and Gaumont (2011) via the preference for quantity principle. Some research attempted to explain many empirical observations, including one that is important, particularly the GDP fluctuations, Khan and Thomas (2007). These authors develop an equilibrium business cycle model where non-convex delivery costs lead firms to follow (S, s) inventory policies, in line with Herbert E. Scarf (1960). Their model reproduces two-thirds of the (pro)-cyclical variability of inventory investment as well as a countercyclical inventory-to-sales ratio and greater volatility in production than sales, see also Schutte (1984) with instantaneous production.

Some authors depart from the argument that the costs and demand shocks are predominant source of fluctuations. To explain the reason of the decrease volatility of real GDP growth in the United States since the early 1980s relative to the prior postwar experience, some research have accentuated on improvement in U.S. monetary policy while others to a reduction in the size of the shocks hitting the U.S. economy. However, Khan, McConnell and Perez-Quiroz (2002) argue that changes in inventory behavior stemming from improvements in information technology (IT) have played a direct role in reducing real output volatility.

In addition to the theoretical and empirical literature proposing a supply-side explana-

tion of the existence of unsold goods through the assumption of uncertain demand (delay between production and sales), this paper attempts to point out the existence of preference for quantity as one of explanation of the stock of goods, or largely in the inventory investment. This based-demand assumption allows us to give clearer explanation of GDP fluctuation. The motivation for such an empirical approach of explaining the emergence of stock of goods through the preference for quantity allows to exhibits its important role of sales and give a deeper explanation of the true cause of economic fluctuations. Contrary to preceding research, this paper seeks to attribute the volatility of economic fluctuations to change in inventories investment via the demand-side based on the preference for quantity principle by fitting the theoretical results obtained in Blot, Cayemitte and Gaumont (2011) with the U.S. data.

The methodology used in this paper to estimate the models stemming from the solution of individuals maximizing problem is the Gauss-Newton algorithm. Since there is a nonlinear relation between dependent and independent variables of these models issued from results obtain in Blot et al. (2011), it is impossible to estimate them by using the traditional linear squares (OLS). To solve the problem of nonlinearity, many research papers and textbooks propose a huge of conditions allowing to linearize the nonlinear models in the neighborhood of a given vector of parameters, Marquart (1963), Gallant (1975), Ratkowsky (1983), Bates and Watts (1988), Hayashi (2000), Greene (2003) and Wooldridge (2002).

Section 2 sums up the theoretical basic model that explains the existence of preference for quantity as one the causes of stock of unsold goods in many markets. Section 3 presents the econometric considerations presenting methodology, while Section 4 reports and analyzes the data used to which the the theoretical model is fitted. The section 4 concludes.

A.2 Theoretical Model

The objective of this section is to resume the simplest possible model capable of showing that a γ -equilibrium exists, and for which the stock of unsold goods is not always zero at

the end of the market period, Blot, Cayemitte and Gaumont (2011). Since the theory relating to cleared markets is well-known, we only investigate equilibria where production exceeds sales. In this version of the model, there is perfect information and certainty and everyone is perfectly rational. There is a single good and its price is perfectly flexible. We assume that the main consumer has a preference for quantity³. Such a reasonable assumption (directly issued from both management and marketing empirical observations) dramatically changes the usual conclusion on market functioning, as will become clearer below. For that reason, we develop hereafter a new way of reasoning.

Consider a firm that sells a single good to a main consumer. Suppose that on the market this main consumer buys a quantity $q \in \mathbb{R}_+^*$ if and only if the quantity of displayed goods is $\bar{q} \in \mathbb{R}_+^*$ such that $\bar{q} \geq q$. Note that in neoclassical equilibria, market solutions satisfy the clearing market condition : $\bar{q} = q$. We concentrate hereafter on the other set of equilibria where $\bar{q} > q$. We show that they exist and study their properties. In these equilibria, if there is an insufficient quantity of displayed goods, no consumer will want to buy anything.

Regarding the results obtained in the theoretical model, the log and CES utility functions are used for the empirical evidence. General developments will be illustrated through two these two functions, since they exhibit very different and interesting results. They also allow us to exhibit the households and the firms' behavior and attempt to fit the solutions of the model with the US data. Different cases will be discussed depending on the production and the stock of unsold goods.

A.2.1 Households

As mentioned above, individuals' preferences are captured by the separable utility function U , which is defined on the consumption set. For the sake of simplicity, we use in the section examples of log-utility and CES-utility functions. Suppose that the utility function

3. There is no range effect or variety effect, since we only consider a single good on the market. Any given individual does not want to buy the last three apples left by the other consumers. He prefers to choose the three apples he wants among the various available apples. Since this is true for all consumers, we capture them in the concept of the main consumer.

U of the consumer is defined by the following log utility function :

$$U_1(\bar{q}, \underline{q}, \gamma) = \ln \underline{q} + \gamma \ln(\bar{q} - \underline{q}), \quad (\text{A.1})$$

where γ is the parameter, which stands for the preference for quantity⁴. As expressed in this utility function, this parameter captures the level of weight that households attribute to a high quantity of goods or the stock of unsold goods ($\bar{c} - \underline{c}$) on both the competitive and monopoly markets.

A more general function using to take into account the households preferences is the Constant Elasticity of Substitution (CES) utility function. The latter is defined as follows⁵ :

$$U_2(\bar{q}, \underline{q}, \gamma) = \underline{q}^\rho + \gamma(\bar{q} - \underline{q})^\rho, \quad \rho \neq 0, \quad (\text{A.2})$$

where the parameter γ still indicates the preference for quantity parameter as the preceding log-utility function.

A.2.1.1 Corner solution

Under the consumer's budget constraint that $p\underline{q} = \Omega$, the log-utility and the CES utility functions give respectively the following optimal demands for $\underline{q}_i^* = \frac{\Omega}{p}$, $i = 1, 2$. Using the

4. Since preferences are invariant with respect to any monotonic transformations of utility, the previous function also encompasses the traditional Cobb-Douglas utility function :

$$W_1(\bar{q}, \underline{q}, \gamma) = \exp [U_1(\bar{q}, \underline{q}, \gamma)] = \underline{q}(\bar{q} - \underline{q})^\gamma.$$

Defining $\gamma = \frac{\beta_1}{\alpha_1}$, we have $W_1(\bar{q}, \underline{q}, \gamma) = \underline{q}(\bar{q} - \underline{q})^{\frac{\beta_1}{\alpha_1}}$, and we have :

$$W_2(\bar{q}, \underline{q}, \gamma) = W_1(\bar{q}, \underline{q})^{\alpha_1} = \underline{q}^{\alpha_1}(\bar{q} - \underline{q})^{\beta_1}.$$

5. The CES utility function can be rewritten as $V_2(\bar{q}, \underline{q}) = (W(\bar{q}, \underline{q}), \gamma)^\rho$:

$$V_2(\bar{q}, \underline{q}, \gamma) = a\underline{q}^\rho + (1 - a)(\bar{q} - \underline{q})^\rho.$$

Defining $\gamma = (1 - a)/a$, V_2 can be transformed into U_2 as follows :

$$U_2(\bar{q}, \underline{q}, \gamma) = \underline{q}^\rho + \gamma(\bar{q} - \underline{q})^\rho, \quad \rho \neq 0.$$

fact that the households's indirect utility function satisfies the following conditions for both function examples :

$$\ln \underline{q}_1^* + \gamma \ln(\bar{q} - \underline{q}_1^*) = v_0, \quad (\text{A.3})$$

and

$$(\underline{q}_2^*)^\rho + \gamma(\bar{q} - \underline{q}_2^*)^\rho = v \quad (\text{A.4})$$

Given the previous corner's solution of the households' maximization problem, we have the following demands for goods \bar{q} related respectively to the log and CES functions :

$$\bar{q}_1^* = \frac{\Omega}{p} + \exp \left[\frac{1}{\gamma} \left(v_0 - \ln \left(\frac{\Omega}{p} \right) \right) \right], \quad (\text{A.5})$$

$$\bar{q}_2^* = \frac{\Omega}{p} + \left[\frac{1}{\gamma} \left[v - \left(\frac{\Omega}{p} \right)^\rho \right] \right]^{\frac{1}{\rho}}. \quad (\text{A.6})$$

Importantly, it is a nice result that in the case of a log utility function, the demand for stock cannot be cleared at all since :

$$S_{t,1} = \exp \left[\frac{1}{\gamma} \left(v_0 - \ln \left(\frac{\Omega}{p} \right) \right) \right]. \quad (\text{A.7})$$

is strictly positive. However, in the CES case, where the stock of unsold goods is given by :

$$S_{t,2} = \left[\frac{1}{\gamma} \left[v - \left(\frac{\Omega}{p} \right)^\rho \right] \right]^{\frac{1}{\rho}}, \quad (\text{A.8})$$

there exists a price that clears the stock, which is far from being optimal. It is given by : $p = \Omega / \exp [v_0]$. Given the level of utility, the price that clears the stock of unsold good during the market period is increasing in the consumer revenue for any γ .

A.2.1.2 Impulse buying

Dealing with the interior solution, the utility functions (A.1) and (A.2) allow us to obtain the following consumer's demand for q_i $i = 1, 2$ given by : $q_1^{**} = \frac{1}{1+\gamma} \bar{q}$, and $q_2^{**} = \frac{1}{1+(\gamma)^{\frac{1}{1-\rho}}} \bar{q}$. Note that in the two previous relations we have respectively : $q_1^{**} < \bar{q}$ and $q_2^{**} < \bar{q}$. By using relations (B.67) and (A.4), the latter result of the consumer's demand for \bar{q}_i $i = 1, 2$, such that :

$$\bar{q}_1^{**} = \frac{1+\gamma}{\gamma^{\frac{\gamma}{1+\gamma}}} \exp \left[\frac{v_0}{1+\gamma} \right]$$

$$\bar{q}_2^{**} = \left[\left(\frac{1+\gamma^{\frac{\rho}{1-\rho}}}{1+\gamma^{\frac{1}{1-\rho}}} \right) v \right]^{\frac{1}{\rho}}$$

Both expressions are independent of price. One can easily state the following condition for $q \in \left[\alpha, \min \left\{ \frac{\Omega}{p}, \bar{q}_i^{**} \right\} \right]$, $i = 1, 2$:

$$\alpha < q_1^{**} < \frac{\Omega}{p} \iff \alpha(1+\gamma) < \bar{q}_1^{**} < (1+\gamma) \frac{\Omega}{p}.$$

$$\alpha < q_2^{**} < \frac{\Omega}{p} \iff \alpha(1+\gamma^{\frac{1}{1-\rho}}) < \bar{q}_2^{**} < (1+\gamma^{\frac{1}{1-\rho}}) \frac{\Omega}{p}.$$

A.2.2 Firms

In this section both the competitive and monopoly market structures are considered. Regarding the preference for quantity, the firm produces a high quantity of goods \bar{q} in order to attract the consumer's demand for goods q .

A.2.3 The competitive price

In this subsection, we present the competitive firm. For all that follows, the cost function is defined by : $TC(\bar{q}) = \frac{1}{2}(\bar{q})^2 - \bar{q}$. In the short run, at the prevailing market price the competitive firm chooses the quantity of displayed goods \bar{q} given in (A.5) (respectively, in (A.6)).

For the corner solution, the price p_{ic}^* , $i = 1, 2$, is given. The rational competitive firm maximizes its profit with respect to the quantity of displayed goods, which is the solution of the following problem : $\max_{\bar{q}} p\bar{q} - \frac{1}{2}\bar{q}^2 + \bar{q}$. The first order condition gives : $\bar{q}^* = p + 1$

The rational competitive firm operates a quantity of displayed goods that matches exactly the consumer's solution, and for a log utility function solves : $\left[p + 1 - \frac{\Omega}{p}\right] \left[\frac{\Omega}{p}\right]^{\frac{1}{\gamma}} = e^{\frac{v_0}{\gamma}}$, for which no simple explicit solution is computable.

For a CES utility function, it is not possible to have an explicit general solution. Under the assumption that both the demand for goods \underline{q} and the quantity of displayed goods \bar{q} are perfect substitutes, that is $\rho \rightarrow 1$, the competitive firm solves : $p^2 - \frac{v_0}{\gamma}p - \frac{1+\gamma}{\gamma}\Omega = 0$, such that we obtain

$$p_{2,c}^* = \frac{v_0 \pm \sqrt{v_0^2 - 4\gamma(1+\gamma)\Omega}}{2\gamma}.$$

In the long run, firms enter the market until the zero-profit condition is reached : $\bar{q}^* = 2(p+1)$ so that the previous prices are divided by 2. Using the interior solution for the competitive firm, we obtain the following prices : $\bar{q}_{1,c}^s = \frac{2}{1+\gamma}p_{1,c}^{**}$, and $\bar{q}_{2,c}^s = \frac{2}{1+\gamma^{\frac{1}{1-\rho}}}p_{2,c}^{**}$.

The γ -equilibrium condition $\bar{q}_1^s = \bar{q}^{**}$ allows us to determine the competitive market price : $p_{1,c}^{**} = 1 + \gamma$. and $p_{2,c}^{**} = 1 + \gamma^{\frac{1}{1-\rho}}$.

A.2.4 The monopoly price

Using the same cost function chosen in the preceding subsection, for the case of the budget-constrained consumer's result, the monopoly prices for both log and CES functions are given by :

$$p_{1,M}^* = \gamma^{\frac{\gamma}{1+\gamma}} \exp\left[-\frac{v_0}{\gamma+1}\right] \Omega,$$

and

$$p_{2,M}^* = \left[\frac{v}{1 + \gamma^{\frac{2\rho}{1-\rho}}}\right]^{\frac{1-\rho}{\rho}} \Omega.$$

In both the log and CES utility functions, the monopoly prices are increasing in the revenue's consumer. When the consumer is budget-constrained the monopoly price is higher for increasing value of the preference for quantity parameter in the interval $]0, \bar{\gamma}]$ for the log-utility function while decreasing when the parameter of preference for quantity increases for the CES-utility function. Thus, regarding the household's interior solution, often called

impulse buyer, the monopoly prices are given by :

$$p_{1,M}^{**} = \frac{(1 + \gamma)^2}{\gamma^{\frac{\gamma}{1+\gamma}}} \exp \left[\frac{v_0}{1 + \gamma} \right]$$

and

$$p_{2,M}^{**} = \left(1 + \gamma^{\frac{1}{1-\rho}} \right) \left[\left(\frac{1 + \gamma^{\frac{\rho}{1-\rho}}}{1 + \gamma^{\frac{1}{1-\rho}}} \right) v \right]^{\frac{1}{\rho}} .$$

The monopoly market price is strictly increasing when the preference parameter for quantity increases. However, the latter result is particularly true for lower level of utility v_0 . For higher utility level the monopolist price is convex and increasing with respect to γ . According to the set of assumptions in the theoretical version of the basic model, it is very important to test this parameter of the preference for quantity in a empirical context. This is the objective of the next section.

A.3 Econometric Considerations

Now we turn to present econometric considerations on the stock of unsold goods. In this regard, we take into account the specification of the solution of maximization problem, which allows to capture the households and firms' behavior. Hence, a set of econometric tools are used so as to facilitate the complexity to due the problem structure. Since the regressions are not linear with respect to their parameters, we need to use a specific approach related to the nonlinearity. The methods commonly used are based on the iterative methods allowing to linearize these nonlinear regressions. In this regard, techniques on how to deal with nonlinear regression are given by Ratkowsky [77] and by Bates and Watts [11]. A more extensive treatment of nonlinear regression methodology is given by Seber and Wild [83].

A.3.1 Nonlinear Regression

This subsection exhibits the empirical impact of the preference for quantity γ on the production level and the inventories investment using the results of the log and CES functions. Using the results obtained by the consumer maximization problem relative to the

previous functions allow to estimate equations (A.5), (A.6), (A.7) and (A.8). In econometric terms, the previous equations can be written as follow :

$$\bar{q}_{1,t} = a_0 + a_1 \left(\frac{\Omega_t}{p_t} \right) + a_2 \exp \left[\frac{1}{\gamma} \left(v_0 - \ln \left(\frac{\Omega_t}{p_t} \right) \right) \right] + \epsilon_{1,t}, \quad t = 1 \dots T, \quad (\text{A.9})$$

$$\bar{q}_{2,t} = b_0 + b_1 \left(\frac{\Omega_t}{p_t} \right) + b_2 \left[\frac{1}{\gamma} \left[v - \left(\frac{\Omega_t}{p_t} \right)^\rho \right] \right]^{\frac{1}{\rho}} + \epsilon_{2,t}, \quad t = 1 \dots T, \quad (\text{A.10})$$

$$s_{1,t} = c_0 + c_1 \exp \left[\frac{1}{\gamma} \left(v_0 - \ln \left(\frac{\Omega_t}{p_t} \right) \right) \right] + \epsilon_{3,t}, \quad t = 1 \dots T, \quad (\text{A.11})$$

$$s_{2,t} = d_0 + d_1 \left[\frac{1}{\gamma} \left[v - \left(\frac{\Omega_t}{p_t} \right)^\rho \right] \right]^{\frac{1}{\rho}} + \epsilon_{4,t}, \quad t = 1 \dots T \quad (\text{A.12})$$

where T is the number of observations and $\epsilon_{i,t}$, $i = 1, \dots, 4$ the error terms. From the macroeconomic point of view, we suppose that households buy output according to the following sales equation :

$$q_t = \frac{\Omega_t}{p_t},$$

where p_t is the nominal price of output in period t . This equation is considered as a fairly standard consumption function, generally assumed without much attention given to its microfoundation. However, for a broad class of utility functions, this equation can be derived directly from household utility function maximization under labor sales constraint, where the income Ω_t may be equal to the sum of wage earned from labor and money transfer from the government as is the case in Eckalbar (1984).

Since in each equation the error term is additive, taking the log of each side of them cannot linearize it in order to estimate the unknown parameters a_0 , a_1 , a_2 , b_0 , b_1 , b_2 , c_0 , c_1 , d_0 , d_1 , v_0 , v and γ by the traditional ordinary linear least squares (OLS). However, nonlinear least squares methods allow us to estimate the parameters without transforming

these variables in the previous regressions. That is the case of the preceding models since the relationship between their dependent variables and regressors are nonlinear.

Let us consider the regression (A.9). In terms of vector notations, it can be rewritten to give the following nonlinear regression :

$$\bar{q} = f(X, \beta) + \epsilon, \quad (\text{A.13})$$

where $\beta = (a_0, a_1, a_2, \alpha, v_0, \gamma)'$ is the vector of parameters to be estimated, ϵ a vector of errors term and f the function of class \mathcal{C}^2 satisfying the nonlinear relationship between the production of goods and the vector of parameters β .

The unknown parameter vector β in the nonlinear regression model can be estimated from the data by minimizing a suitable goodness-of-fit expression with respect to β . Assuming that the error term ϵ_t follows a normal distribution, the least squares estimator for the vector of parameters β is also the maximum likelihood estimator. Then, the likelihood function that represents (A.13) is given by the following function :

$$\mathcal{L}(\beta, \sigma^2) = \frac{1}{\sqrt{(2\pi\sigma^2)^T}} \exp \left\{ -\frac{\sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)]^2}{2\sigma^2} \right\}. \quad (\text{A.14})$$

This function is obtained by taking the product of the density functions of *i.i.d.* normally-distributed data points x_t for $t = 1, \dots, T$. The previous function is maximized by the least squares estimator β^* , which also minimizes the following sum of residuals squares :

$$\mathcal{R}(\beta) = \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)]^2, \quad (\text{A.15})$$

which are generally estimated with the criterion based on nonlinear least squares. The estimators and test statistics employed in nonlinear regressions can be characterized as linear and quadratic forms in the vector ϵ which are apparently similar to those which occur in linear regression, Gallant (1975). In terms of vector notations, (A.15) can be written as follows :

$$\mathcal{R}(\beta) = [\bar{q} - f(\beta)]' [\bar{q} - f(\beta)], \quad (\text{A.16})$$

where $f(\beta) = (f(X_1, \beta), f(X_1, \beta), \dots, f(X_T, \beta))'$ and $X_t = (p_t, \Omega_t)'$ are respectively the vector of value function of each observation and the vector of independent variables.

By differentiating (A.16), we have the following relation :

$$\frac{\partial \mathcal{R}}{\partial \beta} = -2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] \frac{\partial f(X_t, \beta)}{\partial \beta},$$

where $\frac{\partial f(X_t, \beta)}{\partial \beta}$ is defined by the following matrix :

$$\begin{pmatrix} \frac{\partial f(X_1, \hat{\beta})}{\partial \beta_0} & \frac{\partial f(X_1, \hat{\beta})}{\partial \beta_1} & \cdots & \frac{\partial f(X_1, \hat{\beta})}{\partial \beta_k} \\ \frac{\partial f(X_2, \hat{\beta})}{\partial \beta_0} & \frac{\partial f(X_2, \hat{\beta})}{\partial \beta_1} & \cdots & \frac{\partial f(X_2, \hat{\beta})}{\partial \beta_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f(X_T, \hat{\beta})}{\partial \beta_0} & \cdots & \cdots & \frac{\partial f(X_T, \hat{\beta})}{\partial \beta_k} \end{pmatrix}$$

Note that this matrix plays the same role as the well-known matrix X used in the linear regression and quadratic forms. The partial derivatives of the sum of squared deviations allows us to estimate parameters in the nonlinear models. Thus, applying to our nonlinear model the first-order conditions for estimating all the parameters gives the following derivatives with respect to components of β respectively :

$$\frac{\partial \mathcal{R}}{\partial \beta_0} = -2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] = 0. \quad (\text{A.17})$$

$$\frac{\partial \mathcal{R}}{\partial \beta_1} = -2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] \frac{\Omega_t}{p_t} = 0. \quad (\text{A.18})$$

$$\frac{\partial \mathcal{R}}{\partial \beta_2} = -2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] \exp \left[\frac{1}{\gamma} \left(v_0 - \ln \left(\frac{\Omega_t}{p_t} \right) \right) \right] = 0. \quad (\text{A.19})$$

$$\frac{\partial \mathcal{R}}{\partial \beta_2} = -2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] \frac{1}{\gamma} \left[\frac{p_t}{\Omega_t} \right]^{\frac{1}{\gamma}} \exp \left[\frac{v_0}{\gamma} \right] = 0. \quad (\text{A.20})$$

The latter expression gives the derivatives of the production function \bar{q}_t respect to the parameter v_0 which captures the consumer's indifference curve. The following is the derivative of the sum of residuals squares \mathcal{R} respect to the parameter capturing the preference for quantity γ of the households :

$$\frac{\partial \mathcal{R}}{\partial \beta_3} = 2 \sum_{t=1}^T [\bar{q}_t - f(X_t, \beta)] \frac{p_t^{\frac{1}{\gamma}}}{\gamma^2} \left[\frac{\exp v_0}{\Omega_t} \right]^{\frac{1}{\gamma}} \left[v_0 + \ln \left[\frac{p_t}{\Omega_t} \right] \right] = 0. \quad (\text{A.21})$$

Note that the latter partial derivatives are obtained by using the solution of the maximizing consumer's behavior related to the log-utility function. Since the two first terms of the nonlinear regression (A.9) are independent of the parameters γ and v_0 , the three last derivatives (A.19), (A.20) and (A.21) are also true for the equation of stock of unsold goods (A.7).

The estimated asymptotic covariance matrix of the coefficient regression is given by the following formula :

$$V(\hat{\beta}) = S^2(F'F)^{-1},$$

where S is the estimated error variance and $\hat{\beta}$ is a random variable which has a k -dimensional multivariate normal distribution with mean β^* and variance-covariance matrix $S^2(F'F)^{-1}$ in large samples and $(T - k) \frac{S^2}{\sigma^2}$ is independently distributed as a chi-squared variable with $T - k$ degrees of freedom. These conditions will allow us to use some inference in order to estimate a confidence interval for the parameters of the econometric models. Before doing so, it is important to note that the previous equations, which are concerned with the first-order conditions, do not have any explicit solutions. In order to solve this problem, linearized methods will be used. In this case we present the algorithm method using to determine the parameters estimates in the following subsection.

A.3.1.1 The Gauss-Newton Method

The Gauss-Newton approach helps compute nonlinear least squares estimators of the econometric models. It is based on the substitution of the first-order Taylor series expansion of the response function in the formula for the sum of squares for errors, Gallant (1975)

and Marquadt (1963)⁶. Since the function f is of class \mathcal{C}^2 in a neighborhood of β_0 which is not identically null, the first-order Taylor series approximation of the response function is given by :

$$f(X, \beta) \approx f(X, \beta_0) + \frac{\partial f(X, \beta_0)}{\partial \beta} (\beta - \beta_0), \quad (\text{A.22})$$

where $\frac{\partial f(X, \beta_0)}{\partial \beta}$ is the $T \times k$ matrix with elements $\frac{\partial f(x_i, \beta_0)}{\partial \beta_j}$. Replacing (A.22) in the sum of residuals squares (A.15), we have the following approximating sum of residuals squares :

$$\mathcal{R}(\beta) = \left\| \bar{q} - f(X, \beta_0) - \frac{\partial f(X, \beta_0)}{\partial \beta} (\beta - \beta_0) \right\|^2,$$

which can be minimized by linear least squares to give the following values of the parameter :

$$\beta_T = \beta_0 + \left[\left(\frac{\partial f(X, \beta_0)}{\partial \beta} \right)' \left(\frac{\partial f(X, \beta_0)}{\partial \beta} \right) \right]^{-1} \left[\frac{\partial f(X, \beta_0)}{\partial \beta} \right] [\bar{q} - f(X, \beta_0)].$$

Contrary to Nerlove (1963), this paper aims to use the functional forms obtained as the solutions of the theoretical model mentioned in the section 2. Thus, the relations (A.9), (A.10), (A.11) and (A.12) will be estimated directly without imposing constraints concerning the form that satisfies the properties. However, we do not ask any questions about whether the preceding equations are or not from a production function which is really a Cobb-Douglas or CES functional form. This choice is due to the fact that the utility functions are chosen depending on some characteristics. We now turn to present the data and methods used to estimate the nonlinear regressions.

A.4 Data and Empirical Results

This subsection consists in fitting the theoretical model to U.S. quarterly data from the first quarter of 1995 to the their quarter of 2011. To estimate parameters of models describing households and firms' behavior mentioned above, we use data issued from U.

6. In his paper, Donald W. Marquardt (1963) developed a maximum neighborhood method which, in effect, performs an optimum interpolation between the Taylor series method and the gradient method, the interpolation being based upon the maximum neighborhood in which the truncated Taylor series gives an adequate representation of the nonlinear model.

S. Bureau of Economic Analysis. In this regard, this paper employs seasonally adjusted quarterly data extending over the period 1995 to 2011 in order to estimate the nonlinear equations (A.5), (A.6), (A.7) and (A.8). For doing so, we use the gross domestic product (GDP), the production of durable goods, the production of nondurable goods and changes in private inventories as component of the U.S. GDP. The change in private inventories or inventory investment, noted by CIPI in the U.S. National Income and Product Accounts (NIPA), is a measure of the value of change in the physical volume of the inventories or stock of goods that businesses hold to support their production and distribution activities. Used as a proxy of the stock of goods, the CIPI is considered as one of the most volatile components of gross domestic product (GDP) and allows to assess economic fluctuations (Figures A.1 and A.2). To include the value of currently produced goods that are not yet sold and to exclude the value of goods produced in previous periods, change in private inventories must be included in the GDP calculation⁷. Thus, GDP can also be seen as the sum of final sales of domestic product and the change in private inventories (U.S. Bureau of Economic Analysis, 2011).

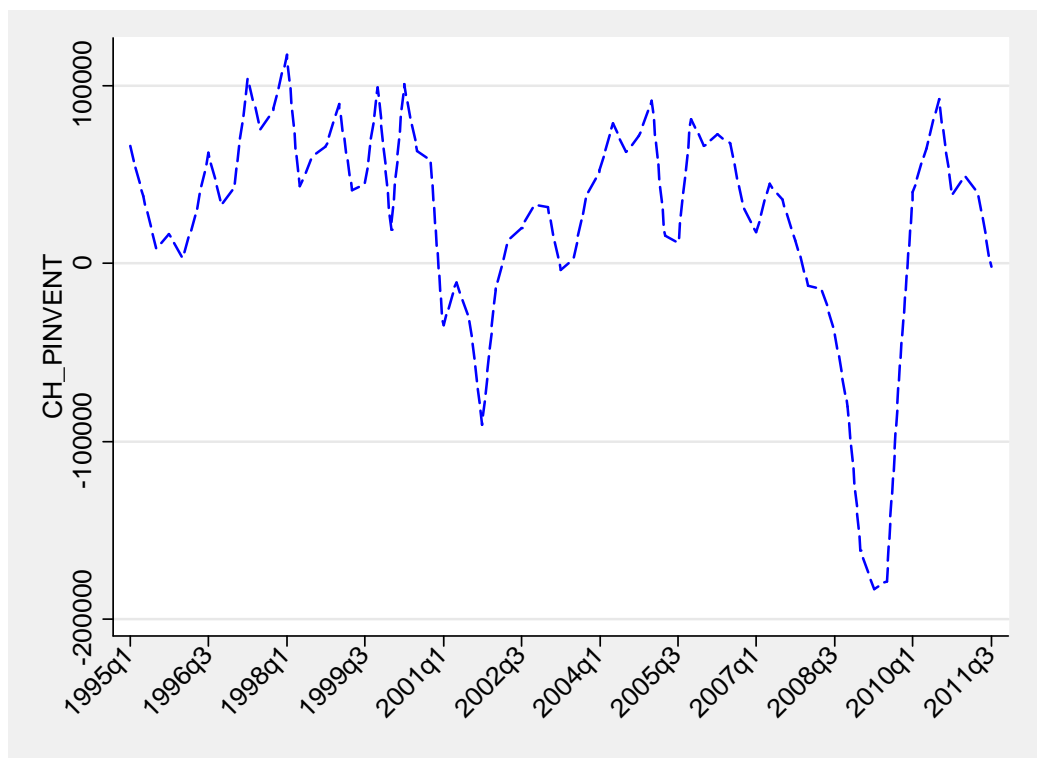
All the previous variables are quarterly and 2005 chain-based⁸. In order to capture the price influence on the production goods and nondurable goods and the inventories level, we use as proxy both the consumer price index of goods and consumer price index of nondurable goods. They are all index numbers, 2005 = 100 and seasonally adjusted. The seasonal adjustment allows us to abstract all repeated events in the series, which are better to be treated regarding the relative preference for quantity parameter. In order to take into account the representative households's revenue effect on the production, we use the data on personal income in Billions of U.S. dollars, issued from U.S. Department of Commerce : Bureau of Economic Analysis.

Contrary to Ramy (1989), we do not use in this document the aggregation procedure

7. In 2011 inventory investment decreased more than in the second quarter and subtracted 1.55 percentage points from real GDP growth after subtracting 0.28 percentage point.

8. Note : Chained (2005) dollar series are calculated as the product of the chain-type quantity index and the 2005 current-dollar value of the corresponding series, divided by 100. Because the formula for the chain-type quantity indexes uses weights of more than one period, the corresponding chained-dollar estimates are usually not additive. The residual line is the difference between the first line and the sum of the most detailed lines.

FIGURE A.1 – Change in US private inventories



despite that the theory presented in Section 2 is concerned with consumer's behavior and firm's behavior. The absence of such assumptions aggregation is due to the fact that all data used in this paper are already aggregated. However, the paper does not depart from the possibility to use the industries data or data stemmed from economic sector even though the latter need some aggregate constraints.

The previous Figure depicts evolution of change in private inventories in millions of US dollars from the first quarter of 1995 to the third quarter of 2011. We can observe through Figure A.1 that the inventories investment fluctuate during the period of study for which there are two periods of recession. The first one is concerned with the four quarters of year 2001, where the stock of goods were in drop to 90.7 billions US dollars in fourth quarter of the same year. This strong decrease is due to the fact that the U.S. economy decreased after the Wall Street Center collapsed during the first quarter of 2001. The second recession is attributed to the U.S. financial crisis started in 2008-2009 and propagated through around the world, particularly the developed European economies, the CIPI dropped 183 billions of U.S. dollars in the second quarter of 2008, where the quarterly real GDP decreased about 2 percent during the same period. The largest decrease of CIPI corresponds to the largest decrease of output in the second quarter of the same year.

The analysis of Figure A.2 exhibits a correlation between the growth rate of U.S. real GDP and the change in real inventories investment. The latter allows us to measure the economic fluctuations. The change in real inventory investment is a good indicator that allows to analyze the economic outlook. Note that most positive values of change in inventories mean that the Gross Domestic Product (GDP) exceeded the sum of the final sales components of GDP in the current period. In this case, the excess production is added to inventories. However, a negative value of CIPI indicates that final sales exceeded production in the current period and that the excess sales are filled by drawing down inventories.

Figure A.3 depicts real inventories to sales ($I - S$) ratio from 1995 to 2011 in U.S.A.. Although we globally observe a decline in $I - S$ ratio during the period, it is important to note some increases, particularly a strong increase in 2008. This inventory to sales ratio increase represents a negative sign for the economy. This often indicates larger financial problems that the industry may be facing. That is the case in 2008, where the I-S ratio increased due to the subprime crisis that affected all American economy. This increase may be explained either by an decrease of net sales for a constant level of inventory during or an increasing of stock of goods while the net sales slow. The observation shows that sales have fallen much faster than the inventory during the recession period. For example, in unadjusted numbers sales were down 5.65 percent from the third to second quarters 2008. At the same period, inventories were down 0.7 percent.

The figure 2 indicates with a weak lag that the pro-cyclical characteristic of the change in inventory investment or stock of goods is explicitly viewed in conjoint evolution of both the GDP growth and the change in the real private inventory. However, the previous figure presenting evolution of the real sales and real inventory to sales ratio depicts a good indication and allows to be comfortable with our analysis. Indeed,

We have indicated that the appropriate approach for estimating our models is the

FIGURE A.2 – Change in inventory and Economic growth

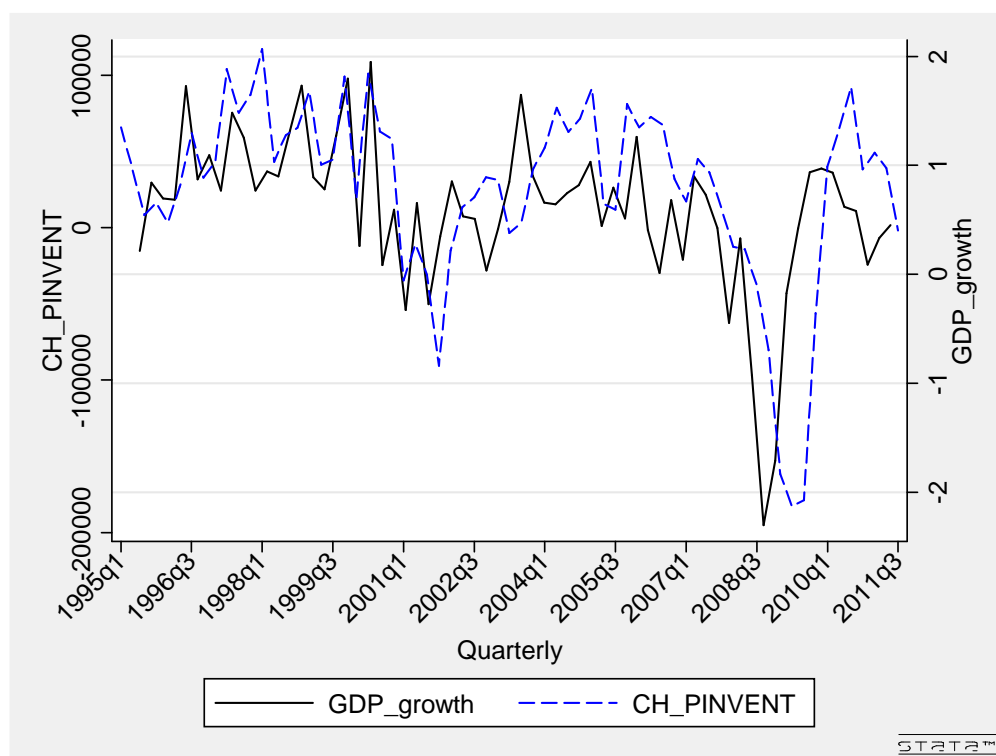


FIGURE A.3 – Inventory to Sales ratio (I-S ratio)

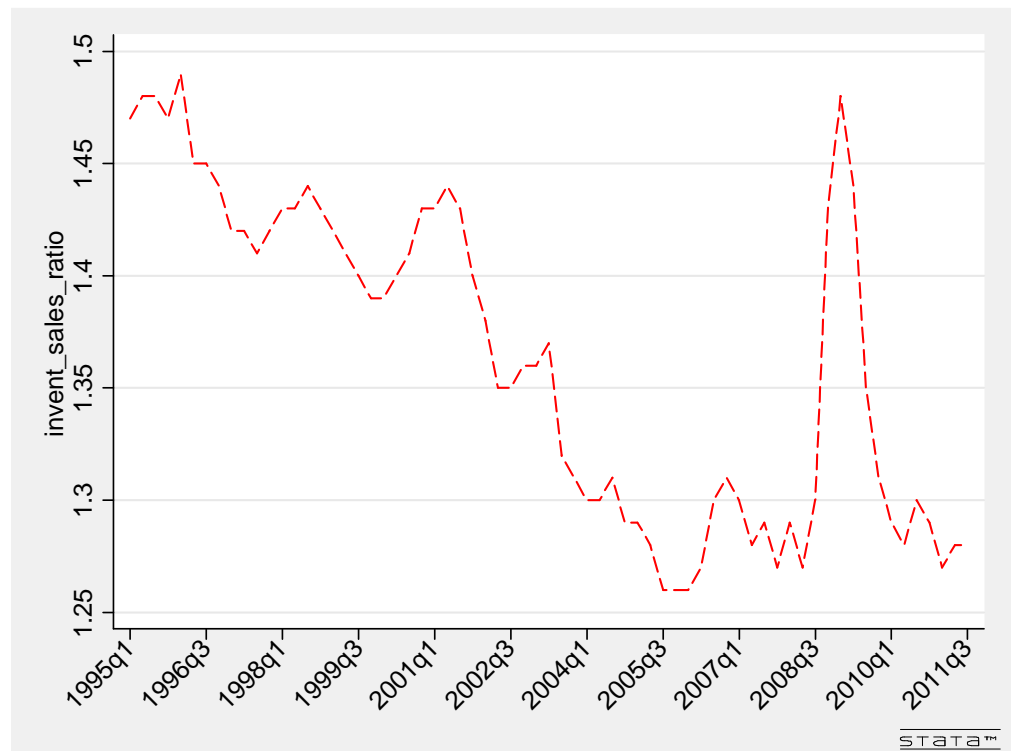
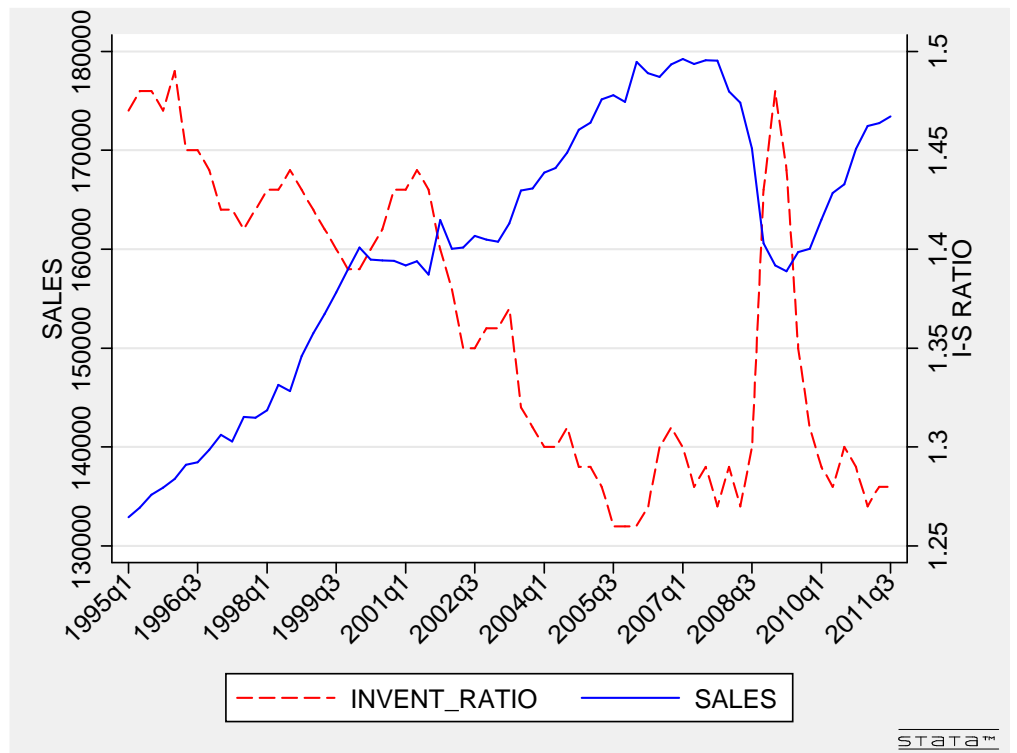


FIGURE A.4 – Real Sales and Inventory to Sales ratio (I-S ratio)



Gauss-Newton method. One important stage of this approach is the choice of the guess values (or initial values) permitting us to conduct the estimation process of the parameters. In this case, it is important to choose reasonable start with initial values in nonlinear least-squares estimation. However, we must experiment with several choices before the problem converges. This procedure is prior to the Gauss method. In this regard, RATS allows us to perform nonlinear estimations in a number of ways, particularly nonlinear least squares (NLLS) and maximum likelihood estimation (ML). The following tables present the main results of the estimation of nonlinear models mentioned above⁹.

Table 1 presents the estimates of the coefficients for the unrestricted nonlinear model (A.9) relative to the logarithmic function. The second row of the table gives the NLLS estimation by the Gauss-Newton algorithm. The estimate of coefficient γ satisfies the theoretical condition that the preference for quantity principle has an impact on the goods production. Indeed, by fitting the model to the U.S. data the results show that the estimate of the preference for quantity parameter is about 0.27 and statistically significant at 0.05 level. It results in an increase of the production of durable goods for any level real personal income, which is considered as a proxy of real sales.

Using the Gauss-Newton algorithm for computing the likelihood estimator, the table 2 depicts that the impact of the preference for quantity principle on the production of nondurable goods is positive and is evaluated about 0.61. Furthermore, it is statistically significant of .05 level. The value of the parameter γ shows the importance of the consumer's preference on the choice of the stock of goods. Furthermore, the results obtained are in line with the theoretical analysis that the production is more volatile than the sales, (table 3 in the appendix). If the real sales increase about one percent then the production of durable goods increases about 3 percent.

9. RATS (Regression Analysis of Time Series) is an econometric software that provides all the basics, including linear and nonlinear least squares, forecasting, SUR, and ARIMA models. But it goes far beyond that, with support for techniques like GMM, ARCH and GARCH models, state space models, and more.

TABLE A.1 – NLLS estimation by the Gauss-Newton Method of the nonlinear regression (A.9)

Parameters of NL Regression	NLLS Estimates	Standard errors	P-value
a_0	4.709088338	1.50608	0.13712423
a_1	1.112392829	11.94378	0.00000000
a_2	-9.996569995	-7.81754	0.00000000
γ	0.273580796	6.49592	0.00000000
v_0	0.964313079	10.36251	0.00000000
$R^2 = 0.993864$			
$F(1, 63) = 0.915590$ with Significance Level : 0.34235199			

TABLE A.2 – NLLS estimation by the Gauss-Newton Method of the nonlinear regressions (A.11)

Parameters of NL Regression	NLLS Estimates	Std errors	Prob> F
c_0	-592.539921	3300.1185	0.8576
c_1	0.000035	0.00027	0.8973
γ	0.616518	0.234533	0.0091
Centered	$R^2 = 0.7883$	$T \times R^2 = 237.667$	
$F(2, 237) = 441.3921$ with Significance Level : 0.00000000			

TABLE A.3 – GNR based on the Restricted NLLS estimation of the nonlinear regressions (A.11)

Parameters of NL Regression	NLLS Estimates	Std errors	Prob> F)
a_0	2.91493523	1.24287831	0.02217392
a_1	0.98663957	0.10048160	0.00000000
a_2	-36.20297281	1.48587256	0.00000000
v_0	0.96431308	0.32670075	0.00000000
Centered $R^2 = 0.991817$			
$F(3, 63) = 2545.2373$ with Significance Level : 0.00000000			

A.4.1 Restricted Model

Trinity test The trinity encompasses the main tests such as the Wald Test (for coefficients restriction), the Likelihood Ratio test and the Lagrange Multiplier. These tests allow us to impose restrictions, Hayashi (2000), Greene (2003) and Wooldridge (2002). First, the Wald test allows to test restrictions for which some coefficients are constant. It can be used to test equality between two coefficients vectors. This paper focuses on the test that the coefficient of preference for quantity γ is equal to one. In this case, we estimate a GNR based on the restricted NLS regression. The following table reports the statistics of the restricted model.

Likelihood ratio test The basic equation is given by : $LR = 2(\log L(\hat{\beta}) - \log L(\tilde{\beta}))$, which is distributed according to $\chi^2_{k_r}$, where $\hat{\beta}$ is the unrestricted estimator of β , $\tilde{\beta}$ the restricted NLS estimate and k_r the number of restrictions imposed by the null hypothesis, Buse (1983). Hence, the construction of the LR statistic needs both the unrestricted and restricted estimates.

Lagrange Multiplier test Lagrange multiplier test is one of the trinity tests, allowing to test a restricted model under the null hypothesis. The LM tests procedure consists in testing the significance of a regression of the residuals on the derivatives of the residuals with respect to all parameters, evaluated at the restricted estimates. In the first case, the hypothesis are stated with respect to the restriction on the related parameters. Since the

Gauss-Newton regressions (GNR) based on the restricted model are estimated, we compute the gradient of the expended model evaluated at the restricted model or the restricted NLS parameters. Consider equation (A.11), we have the following restriction hypothesis :

$$H_0 : s = x(a_0, a_1, v_0, 1) + \epsilon,$$

$$H_1 : s = x(a_0, a_1, v_0, \gamma) + \epsilon,$$

where ϵ is asymptotically normally distributed with zero mean and the variance-covariance matrix $\sigma^2 I$. As in Baltagi (2008), denoting by $\tilde{\beta}$ the restricted NLS estimator of β , the Gauss-Newton regression evaluated at this restricted NLS estimator is given by :

$$(s - \tilde{x}) = \nabla x(\tilde{\beta})b + u,$$

where u is the residuals vector of the GNR. By developing the gradient, we have the following linear regression with respect to the restricted nonlinear least squares estimator :

$$(s - x(\tilde{\beta})) = \sum_{i=1}^4 \frac{\partial x}{\partial \beta_i}(\tilde{\beta}).b_i + u,$$

with :

$$\frac{\partial x}{\partial \beta_0}(\tilde{\beta}) = 1,$$

$$\frac{\partial x}{\partial \beta_1}(\tilde{\beta}) = e^{\tilde{v}_0} \frac{p_t}{\Omega_t},$$

$$\frac{\partial x}{\partial \beta_2}(\tilde{\beta}) = \tilde{a}_1 e^{\tilde{v}_0} \frac{p_t}{\Omega_t},$$

$$\frac{\partial x}{\partial \beta_3}(\tilde{\beta}) = \tilde{a}_1 e^{\tilde{v}_0} \frac{p_t}{\Omega_t} \left[1 + \ln \left(\frac{p_t}{\Omega_t} \right) \right].$$

Since the derivatives with respect to a_0 is constant and those with respect to a_1 and v_0 are linearly dependent, we have to create two new series $\frac{\partial x}{\partial \beta_2}(\tilde{\beta})$ and $\frac{\partial x}{\partial \beta_3}(\tilde{\beta})$ so as to build the linear regression of restricted model residuals on all variables including a constant by OLS.

Thus, the OLS estimates obtained from the previous regression allow to compute the LM -statistic, which is equivalent to TR_{un}^2 such that R_{un}^2 is the uncentered R -squared. Under the null hypothesis and homoskedasticity, we assume that LM is distributed according to $\chi_{k_r}^2$, where k_r is the number of restricted parameters. In this paper, the number $k_r = 1$ means that there is only one parameter which is imposed to be equal to one, say the parameter γ .

A.5 Conclusion

The aim of this paper was to fit the model stemmed from the results of the theoretical model with flexible price to the U.S. data from the first quarter of 1995 to the their quarter of 2011. Since the model is nonlinear in the parameters, the Gauss-Newton regression (GNR) was used to estimate the model. Results obtained show that the impact of preference for quantity principle on the inventory investment or the stock of goods is significantly different from zero. They also have confirmed regularities that the production exceed sales since and co-move.

The paper has analyzed the conjoint evolution of change in inventory (CIPI) and the GDP growth rate, and in the second hand, it has pointed out contrasted evolution of the real sales with respect to the inventory to sales ratio ($I - S$). The latter is considered as an indicator of economic fluctuation. The positivity of $I - S$ presents bad sign for the economy. This is case of the U.S. economy, where the it attained the highest level in 2008 related to the subprime crisis.

A.6 Appendix

A.6.1 CES function

Regarding the constant elasticity of Substitution (CES) function, we assume that the residuals squares of its econometric formulation is given by

$$\mathcal{R}_1(\beta) = \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right]^2, \quad (\text{A.23})$$

where the function g is assumed to be twice continuously differentiable on the parameters space and states a nonlinear relationship between the production variable \bar{q} and the independent variables. Using (A.23) we get the following partial derivatives of the residuals squares \mathcal{R}_1 with respect to the respective vector of parameters $\beta = (b_0, b_1, b_2, v, \gamma, \rho)'$:

$$\frac{\partial \mathcal{R}_1(\beta)}{\partial \beta_0} = -2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] = 0, \quad (\text{A.24})$$

$$\frac{\partial \mathcal{R}_1(\beta)}{\partial \beta_1} = -2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] \frac{\Omega_t}{p_t} = 0, \quad (\text{A.25})$$

$$\frac{\partial \mathcal{R}_2(\beta)}{\partial \beta_2} = -2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] \left[\frac{1}{\gamma} \left[v - \left(\frac{\Omega_t}{p_t} \right)^\rho \right] \right]^{\frac{1}{\rho}} = 0, \quad (\text{A.26})$$

$$\frac{\partial \mathcal{R}_2(\beta)}{\partial \beta_3} = 2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] \frac{1}{\gamma^2 \rho} \left[v - \left(\frac{\Omega_t}{p_t} \right)^\rho \right] \left[\frac{v - \left(\frac{\Omega_t}{p_t} \right)^\rho}{\gamma} \right]^{\frac{1-\rho}{\rho}} = 0, \quad (\text{A.27})$$

$$\frac{\partial \mathcal{R}_2(\beta)}{\partial \beta_4} = -2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] \frac{1}{\gamma \rho} \left[\frac{v - \left(\frac{\Omega_t}{p_t} \right)^\rho}{\gamma} \right]^{\frac{1-\rho}{\rho}} = 0, \quad (\text{A.28})$$

$$\frac{\partial \mathcal{R}_2(\beta)}{\partial \beta_2} = 2 \sum_{t=1}^T \left[\bar{q}_{2,t} - g(X_t, \beta) \right] \left[\frac{v - \left(\frac{\Omega_t}{p_t} \right)^\rho}{\gamma} \right]^{\frac{1}{\rho}} \left[\frac{\left(\frac{\Omega_t}{p_t} \right)^\rho \ln \left(\frac{\Omega_t}{p_t} \right)}{\gamma \left(v - \left(\frac{\Omega_t}{p_t} \right)^\rho \right)} - \frac{\ln \left(\frac{v - \left(\frac{\Omega_t}{p_t} \right)^\rho}{\gamma} \right)}{\rho^2} \right] = (\text{A.29})$$

A.6.2 Testing Trinity

Using the NL -regression (A.12), the hypothesis formulation is given by the following :

$$H_0 : s_2 = x_2(d_0, d_1, v_0, 1, \rho) + \epsilon_4,$$

$$H_1 : s_2 = x_2(d_0, d_1, v_0, \gamma, \rho) + \epsilon_4,$$

where ϵ_4 is asymptotically normally distributed with zero mean and the variance-covariance matrix $\sigma^2 I$. Hence, the GNR evaluated at the restricted NLS estimator $\hat{\beta}$ of the vector of parameters β , we have the following L-regression :

$$(s_2 - x_2(\hat{\beta})) = \sum_{j=1}^5 \frac{\partial x_2}{\partial \beta_j}(\hat{\beta}) \cdot b_j + \mathcal{R}_4,$$

where

$$\frac{\partial x_2}{\partial \beta_0}(\hat{\beta}) = 1,$$

$$\frac{\partial x_2}{\partial \beta_1}(\hat{\beta}) = \frac{\partial x_2}{\partial d_1}(\hat{\beta}) = \left[\hat{v}_0 - \left(\frac{\Omega}{p}\right)^{\hat{\rho}} \right]^{1/\hat{\rho}},$$

$$\frac{\partial x_2}{\partial \beta_2}(\hat{\beta}) = \frac{\partial x_2}{\partial v_0}(\hat{\beta}) = \frac{\hat{d}_1}{\hat{\rho}} \left[\hat{v}_0 - \left(\frac{\Omega}{p}\right)^{\hat{\rho}} \right]^{(1-\hat{\rho})/\hat{\rho}},$$

$$\frac{\partial x_2}{\partial \beta_3}(\hat{\beta}) = \frac{\partial x_2}{\partial \gamma}(\hat{\beta}) = -\frac{\hat{d}_1}{\hat{\rho}} \left[\hat{v}_0 - \left(\frac{\Omega}{p}\right)^{\hat{\rho}} \right]^{1/\hat{\rho}},$$

$$\frac{\partial x_2}{\partial \beta_4}(\hat{\beta}) = \frac{\partial x_2}{\partial \rho}(\hat{\beta}) = -\frac{\hat{d}_1(\hat{v}_0 - (\frac{\Omega}{p})^{\hat{\rho}})}{\hat{\rho}^2} \left[\hat{\rho} \left(\frac{\Omega}{p}\right)^{\hat{\rho}} \ln\left(\frac{\Omega}{p}\right) + (\hat{v}_0 - (\frac{\Omega}{p})^{\hat{\rho}}) \ln(\hat{v}_0 - (\frac{\Omega}{p})^{\hat{\rho}}) \right]^{1/\hat{\rho}}.$$

Since the derivatives of the residuals with respect to d_0 and d_1 are respectively constant and $\left[\hat{v}_0 - \left(\frac{\Omega}{p}\right)^{\hat{\rho}} \right]^{1/\hat{\rho}}$, we create the following new series $\frac{\partial x_2}{\partial v_0}(\hat{\beta})$, $\frac{\partial x_2}{\partial \gamma}(\hat{\beta})$ and $\frac{\partial x_2}{\partial \rho}(\hat{\beta})$ in order to conduct the linear regression.

Auxiliary Regression LM Test An auxiliary regression test is a secondary regression which includes some variables generated from your primary regression, typically a function

of the residuals. The actual test is usually either for the regression having a zero R^2 or for some regressors having zero coefficients. The R^2 test may be based upon either the centered R^2 or the uncentered one.

A.6.3 Estimation Results

FIGURE A.5 – GDP growth rate and Inventories to sales ratio from 1995 :1 to 2011 :3

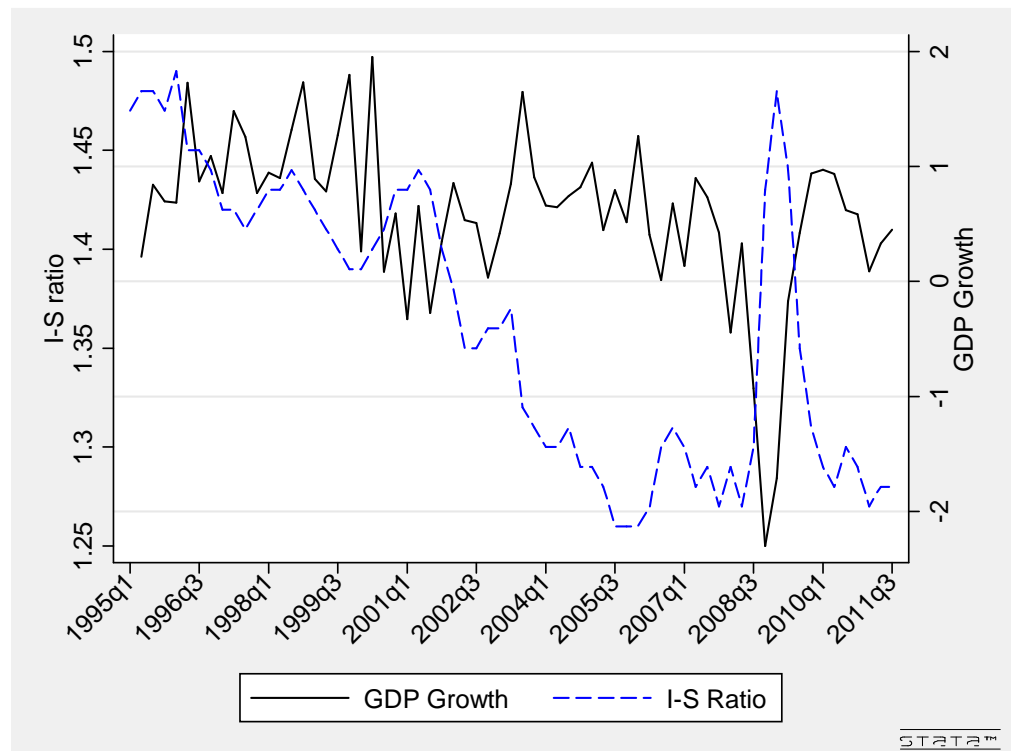


TABLE A.4 – Results of GNR Estimation of the stock of US Unsold goods

```

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 72 Iterations. Final criterion was 0.0000031 <= 0.0000100
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 64
Centered R**2 0.613664 R Bar **2 0.601591
Uncentered R**2 0.988848 T x R**2 66.253
Mean of Dependent Variable 2729.1865672
Std Error of Dependent Variable 474.0757045
Standard Error of Estimate 299.2349017
Sum of Squared Residuals 5730657.6903
Log Likelihood -475.51660
Durbin-Watson Statistic 0.106439

Variable Coeff Std Error T-Stat Signif
*****
1. A0 0.558746423 0.133773412 4.17681 0.00009124
2. A1 -2.731641994 0.000000000 0.00000 0.00000000
3. A2 0.309898741 0.030154613 10.27699 0.00000000

t(64)= -22.885429 or F(1,64)= 523.742882 with Significance Level 0.00000000

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 11 Iterations. Final criterion was 0.0000000 <= 0.0000100
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 65
Centered R**2 0.312500 R Bar **2 0.301923
Uncentered R**2 0.980155 T x R**2 65.670
Mean of Dependent Variable 2729.1865672
Std Error of Dependent Variable 474.0757045
Standard Error of Estimate 396.0950543
Sum of Squared Residuals 10197933.985
Log Likelihood -494.82449
Durbin-Watson Statistic 0.010578

Variable Coeff Std Error T-Stat Signif
*****
1. A0 0.000348873 0.000006175 56.49829 0.00000000
2. A1 -4.644995405 0.000000000 0.00000 0.00000000

Chi-Squared(1)= 38.615775 with Significance Level 0.00000000

Linear Regression - Estimation by Least Squares
Dependent Variable RESIDS
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 64
Centered R**2 0.588662 R Bar **2 0.575808
Uncentered R**2 0.588961 T x R**2 39.460
Mean of Dependent Variable -10.5118375
Std Error of Dependent Variable 392.9401714
Standard Error of Estimate 255.9220422
Sum of Squared Residuals 4191749.8679
Regression F(2,64) 45.7950
Significance Level of F 0.00000000
Log Likelihood -465.04074
Durbin-Watson Statistic 0.126126

Variable Coeff Std Error T-Stat Signif
*****
1. Constant -204658.2103 45340.7629 -4.51378 0.00002793
2. P 11066.0088 2519.3862 4.39234 0.00004300
3. A2DERIV 421.5568 96.5667 4.36545 0.00004728

Chi-Squared(1)= 39.460379 with Significance Level 0.00000000

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 3 Iterations. Final criterion was 0.0000000 <= 0.0000100
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 64
Centered R**2 -0.690605 R Bar **2 -0.743436
Uncentered R**2 0.999098 T x R**2 66.940
Mean of Dependent Variable 7.8957390452
Std Error of Dependent Variable 0.1838517332
Standard Error of Estimate 0.2427564174
Sum of Squared Residuals 3.7715634049
Log Likelihood 1.31742
Durbin-Watson Statistic 0.010018

Variable Coeff Std Error T-Stat Signif
*****

```

TABLE A.5 – Results of GNR Estimation of the production

```

Linear Regression - Estimation by Least Squares
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations      67      Degrees of Freedom      65
Centered R**2      0.914707      R Bar **2      0.913395
Uncentered R**2      0.999961      T x R**2      66.997
Mean of Dependent Variable      13.709929057
Std Error of Dependent Variable      0.293682215
Standard Error of Estimate      0.086426820
Sum of Squared Residuals      0.4855236861
Regression F(1,65)      697.0828
Significance Level of F      0.00000000
Log Likelihood      69.99298
Durbin-Watson Statistic      0.090291

Variable      Coeff      Std Error      T-Stat      Signif
*****
1. Constant      -25.18317357      1.47313164      -17.09499      0.00000000
2. S_A      3.24616068      0.12294980      26.40233      0.00000000

Nonlinear Least Squares - Estimation by Simplex
NO CONVERGENCE IN 528 ITERATIONS
LAST CRITERION WAS 0.0009433
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations      67      Degrees of Freedom      62
Centered R**2      0.979407      R Bar **2      0.978079
Uncentered R**2      0.999991      T x R**2      66.999
Mean of Dependent Variable      13.709929057
Std Error of Dependent Variable      0.293682215
Standard Error of Estimate      0.043482039
Sum of Squared Residuals      0.1172226387
Log Likelihood      117.60161
Durbin-Watson Statistic      0.360753

Variable      Coeff
*****
1. A0      15.30367435
2. A1      0.00021917
3. A2      -37.57751104
4. A3      0.96431308
5. A4      1.12882304

```

TABLE A.6 – Results of GNR Estimation of the production (continued)

```

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 95 Iterations. Final criterion was 0.0000038 <= 0.0000100
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 62
Centered R**2 0.993864 R Bar **2 0.993468
Uncentered R**2 0.999997 T x R**2 67.000
Mean of Dependent Variable 13.709929057
Std Error of Dependent Variable 0.293682215
Standard Error of Estimate 0.023736149
Sum of Squared Residuals 0.0349310970
Regression F(4,62) 2510.4148
Significance Level of F 0.000000000
Log Likelihood 158.15998
Durbin-Watson Statistic 0.496942
    
```

Variable	Coeff	Std Error	T-Stat	Signif
1. A0	4.709088338	3.126713452	1.50608	0.13712423
2. A1	1.112392829	0.093135773	11.94378	0.00000000
3. A2	-9.996569995	1.278736420	-7.81754	0.00000000
4. A3	0.964313079	0.093057867	10.36251	0.00000000
5. A4	0.273580796	0.042115387	6.49592	0.00000000

t(62)= 0.956865 or F(1,62)= 0.915590 with Significance Level 0.34235199

```

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 2 Iterations. Final criterion was 0.0000001 <= 0.0000100
Dependent Variable Q
Quarterly Data From 1995:01 To 2011:03
Usable Observations 67 Degrees of Freedom 63
Centered R**2 0.991817 R Bar **2 0.991427
Uncentered R**2 0.999996 T x R**2 67.000
Mean of Dependent Variable 13.709929057
Std Error of Dependent Variable 0.293682215
Standard Error of Estimate 0.027191964
Sum of Squared Residuals 0.0465823839
    
```

```

Regression F(3,63) 2545.2373
Significance Level of F 0.000000000
Log Likelihood 148.51717
Durbin-Watson Statistic 0.377478
    
```

Variable	Coeff	Std Error	T-Stat	Signif
1. A0	2.91493523	1.24287831	2.34531	0.02217392
2. A1	0.98663957	0.10048160	9.81911	0.00000000
3. A2	-36.20297281	1.48587256	-24.36479	0.00000000
4. A3	0.96431308	0.32670075	0.15982	0.00000000

TABLE A.7 – Results of the stock of goods (model with the Log utility function)

```

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 70 Iterations. Final criterion was 0.0000000 <= 0.0000100
Dependent Variable S
Monthly Data From 1992:01 To 2011:12
Usable Observations 240 Degrees of Freedom 237
Centered R**2 0.788352 R Bar **2 0.786566
Uncentered R**2 0.990279 T x R**2 237.667
Mean of Dependent Variable 8648.2916667
Std Error of Dependent Variable 1901.4471032
Standard Error of Estimate 878.4479684
Sum of Squared Residuals 182885987.45
Regression F(2,237) 441.3921
Significance Level of F 0.00000000
Log Likelihood -1965.79340
Durbin-Watson Statistic 0.042493

Variable Coeff Std Error T-Stat Signif
*****
1. A0 -592.539921 3300.118504 -0.17955 0.85765841
2. A1 0.000035 0.000270 0.12920 0.89730649
3. A2 0.616518 0.234533 2.62871 0.00913130

t(237)= -1.635093 or F(1,237)= 2.673529 with Significance Level 0.10335703

Nonlinear Least Squares - Estimation by Gauss-Newton
Convergence in 2 Iterations. Final criterion was 0.0000002 <= 0.0000100
Dependent Variable S
Monthly Data From 1992:01 To 2011:12
Usable Observations 240 Degrees of Freedom 238
Centered R**2 0.787283 R Bar **2 0.786389
Uncentered R**2 0.990230 T x R**2 237.655
Mean of Dependent Variable 8648.2916667
Std Error of Dependent Variable 1901.4471032
Standard Error of Estimate 878.8115117
Sum of Squared Residuals 183809702.18
Regression F(1,238) 880.8579
Significance Level of F 0.00000000
Log Likelihood -1966.39796
Durbin-Watson Statistic 0.039840

Variable Coeff Std Error T-Stat Signif
*****
1. A0 -5960.416937 495.477475 -12.02964 0.00000000
2. A1 0.094117 0.003171 29.67925 0.00000000

Chi-Squared(1)= 1.209133 with Significance Level 0.27150381

```


B A Two-country Overlapping Generations Model with Stock of unsold goods and Liquidity Constraint

This paper analyzes a two-country overlapping generations (OLG) model based on the principle of preference for quantity. Such a principle allows us to extend the dynamic OLG equilibrium to a frictionless dynamic OLG equilibrium, which is compatible with stock of unsold goods. Under constraint imposed by the cash-in-advance and the level of preference for quantity in the first the period of lifetime, this chapter also accounts for the steady-state welfare implications of stock of capital mobility in an open economy and the effect of monetary policy on the γ -equilibrium.

JEL Classification : O4, E4, F2.

Key words : Stock of unsold Goods, Cash-in-Advance, Overlapping Generations, Capital Mobility, Monetary policy.

B.1 INTRODUCTION

Based on the management science concept of facings, the concept of preference for quantity principle is introduced into the overlapping generations (OLG) economy¹. Such a principle explains the emergence and persistence of stock of unsold goods as an equilibrium phenomenon with perfect information, no uncertainty, price flexibility and full rationality of both consumers and firms. It allows to extend the dynamic OLG equilibrium to a frictionless dynamic OLG γ -equilibrium. Such an equilibrium is compatible with stocks of unsold goods and contains as a sub-case the neoclassical equilibria. In the static version, Blot, Cayemite and Gaumont (2011) have showed how such a stock of unsold goods emerges in the economy. Application of the preference for quantity principle on labor market functioning leads to an explication of the emergence of unemployment (as a stock of workers), without the help of fixed wage or imperfect information².

Pioneered by Maurice Allais (1947), Paul Samuelson (1958) and Peter Diamond (1965), overlapping generations models generated a huge literature. Most of these research are based on the capital and labor mobility in a competitive OLG economy, Buitier (1981), Galor (1986), Crettez, Michel and Wignolle (1998), etc.. while early works accounted for international trade in capital goods, in consumer goods (Fischer and Frenkel (1972)) and in financial claims as in Borts (1964). In a context of dynamic general equilibrium, this model allows not only to analyze international labor migration but exhibits in a two-sector economy, the existence and global stability of a unique perfect-foresight equilibrium as in with Galor (1986, 1992). Farmer and Wendner (2003) are in line with previous author, except that they seek to characterize the existence and stability properties of steady-state solutions as well as the nature of transition paths of a two-sector growth model by comparing the properties of a Cobb-Douglas-Leontieff economy with heterogeneous capital.

1. The preference for quantity captures the consumer's valuation in terms of utility of the available quantity of the displayed good he decides not to buy.

2. Note that we use the concept of Principle, and not the one of Assumption. Indeed, an assumption refers to the act of taking something for granted or something that is taken for granted, whereas a principle is a general and fundamental truth that may be used in deciding conduct or choice, it is a rule of action. For example, the Archimed Principle is not an assumption exactly as the minimization of energy principle is not an assumption in physics, the maximization of utility or profit is not an assumption in economics but a principle in itself.

However, studies of gains from free trade in OLG model are early referred to works of Kemp (1962), Samuelson (1962), Grandmont and McFadden (1972), Grossman (1984) and Kemp and Wong (1995).

However, a bulk of papers account for capital mobility. From differences in time preference, Buiter (1981) seeks to explain international capital movements and to evaluate the welfare implications of a change from trade and financial autarky. In their paper, Crettez, Michel and Vidal (1996) have studied the pattern of capital mobility in a two-country overlapping generations world in which production uses three inputs capital, labor and land. Although dealing with the capital mobility within a two-country, the paper used a dynastic model in which each individual has a degree of altruism toward children, Vidal (2000). Our work is line with that of previous authors the fact that this paper considers the quasi-mobility of capital across countries. Even though the preference for quantity principle, which is considered as one of important assumptions of paper traps the OLG economy with stock of unsold goods, we only consider capital movements of capital across countries.

However our paper deals with mobility of capital in a two-country world, where each individual born in country 1 as well as in country 2 has a preference for quantity. Regarding this preference acted in the first period of lifetime, the paper departs as little as possible from the traditional paradigm that all markets are in equilibrium according to Walras' Law (Lange, 1942). Under both the traditional paradigm, in which all goods are priced at their equilibrium value, there is no stock of unsold goods. Indeed, with perfect information, pure rationality and price flexibility, firms optimize their profit, whereas consumers maximize their utility function under budget constraints, Allais (1947), Samuelson (1958), Diamond (1965) and De La Croix and Michel (2002). However, to take into account this new concept of preference the contrapositive of Walras' Law is emphasized. The central difference between the traditional models and the model developed in this paper is that our OLG model accounts for the preference for quantity of individuals in each country. The latter allows to extend the traditional equilibrium to a new equilibrium with stock of unsold goods.

Despite that the economic literature proposes various ways to account for unsold goods in economics, there are no works on overlapping generations models dealing with the stock of unsold goods. The motivation for a deterministic explanation of the emergence of unsold goods is the following. First, this paper shows that with existence of money, the local stability of the autarkic steady-state γ -equilibrium depends on the level preference for quantity. The latter allows us to discuss briefly how consumers are willingness to care about current consumption relatively to future consumption. This is new since the consumer's choice depends not only on his impatience to consume and the rate of return on savings and money holding, but on his preference for quantity.

The present paper investigates the steady-state γ -equilibrium in an autarkic economy trapped by preference for quantity. When the latter does not hold, our model coincides with the traditional steady-state equilibrium in OLG model. Otherwise, when the assumption of autarky is relaxed the paper shows that with respect to cash in advance constraint and the preference for quantity acted in the first period of individuals lifetime and the perfect-foresight anticipations on capital invested abroad, the steady-state welfare is increasing in the domestic demand for capital. It is important to note that contrary to the traditional literature the consumers' problem prints out a new inter-temporal marginal rate of substitution which is not equal to one. Traditionally, individuals do not like the intertemporal consumption shocks. In this case they allocate their wage in order to guarantee a consumption smoothing between the first and second period of their lifetime. Furthermore, our OLG model studies the impact of monetary policy on dynamics of capital and on individuals' behavior in one hand and the Phillips curve in an open OLG economy.

The paper is organized as follows. Section 2 presents a two-country OLG model with cash in advance. Section 3 outlines the autarkic steady-state γ -equilibrium, while section 4 is concerned with steady-state welfare implications with stock of unsold goods and an extension to a two-country OLG with CES utility function and a Cobb-Douglas production. Section 5 deals with assessment of the impact of the monetary policy on the dynamics of capital when the preference for quantity principle is acting, prior to Section 6 which concludes.

B.2 THE MODEL

Consider a perfectly competitive international world with two countries, 1 and 2, where economic activity in each country is operated over infinite discrete time, such that $t = 0, 1, 2, \dots, \infty$. In every period, a new generation of individuals N_t is born in country 1 according to the following law of motion $N_{t+1} = (1 + n)N_t$ where $N_0 > 0$ is given, and $n > -1$ is the same rate of population growth in each country. The total supply of labor in country 1 is L_t . In each country, a single tradable good is produced using two factors of production, labor L_t and capital K_t . Capital fully depreciates after one period. Denoting $k_t = K_t/L_t$ the capital per worker and $k_t^s = K_t^s/N_t$ the capital per young. Unrestricted as well as restricted capital mobility are considered. According to the gap between domestic and foreign rates of return on capital, we assume that there is a quasi-mobility of capital across countries. Indeed the flow of capital from one country to another is stated under a constraint. In this regard, individuals born in period t in country 1 can not invest all their savings in the other country. For the sake of simplicity, the variables of country 2 is superscripted by symbol (\star) while those of country 1 by absence of superscript.

B.2.1 *Households*

Individuals are identical within as well as across generations and live two periods. In the first period, they supply one unit of labor and earn the competitive nominal market wage W_t when young. In the second period they are retired. Recall that the consumer's preference for quantity principle states that in order to consume \underline{c}_t a consumer needs to face \bar{c}_t unit of goods in period t , with $\underline{c}_t \leq \bar{c}_t$. When old, they consume d_{t+1} unit of goods. We assume no bequests in both countries. Contrary to Vidal (2000), this paper does not deal with any altruism from one generation to another.

Let α, β be two real numbers such that $0 < \alpha < \beta < \infty$.³ Let $I := [\alpha, \beta]$.

3. Note that α is analogous to s and β to S in the (s, S) model, and that s exactly corresponds to the

Assumption 10. Suppose that consumer's preferences are represented by the function

$$\begin{aligned} U : \mathcal{D}(U) &\rightarrow [-\infty, \infty) \\ (\bar{c}, \underline{c}, d, \gamma) &\mapsto U(\bar{c}, \underline{c}, d, \gamma), \end{aligned}$$

where $\mathcal{D}(U) = \{(\bar{c}, \underline{c}, d, \gamma) \in I \times I \times \mathbb{R}_+^* \times [\underline{\gamma}, \bar{\gamma}] : \bar{c} \geq \underline{c} \geq 0, d \geq 0\}$ is the domain of U and γ is a parameter which captures the consumer's preference for quantity.

Definition 6. We call **main consumer** any consumer who has a positive preference for quantity, i.e. $\underline{\gamma} > 0$. We call **residual consumer** any consumer who has no preference for quantity, i.e. $\underline{\gamma} = \bar{\gamma} = \gamma = 0$.

Property 1. The quantity of goods \bar{c}_t depends exclusively on the preference for quantity principle stated at the first period of the individuals lifetime. Hence, if $\underline{\gamma} = \bar{\gamma} = \gamma = 0$ then the inter-temporal utility function becomes only a function of the first and second period consumption goods \underline{c}_t and d_{t+1} respectively.⁴

Note that young individuals are main consumers while old individuals are residual consumers.

Denoting the interior of $\mathcal{D}(U)$ relatively to the topology of $I \times I \times \mathbb{R}_+^* \times [\underline{\gamma}, \bar{\gamma}]$ by

$$\mathcal{D}^\circ(U) = \{(\bar{c}, \underline{c}, d, \gamma) \in I^\circ \times I^\circ \times \mathbb{R}_+^* \times [\underline{\gamma}, \bar{\gamma}] : \bar{c} > \underline{c} > 0, d > 0\},$$

we get the following assumption.

Assumption 11. For all $(\bar{c}, \underline{c}, d, \gamma) \in \mathcal{D}^\circ(U)$, $U(\bar{c}, \underline{c}, d, \gamma) \in \mathbb{R}$.

If $\bar{c} \in I$, then $\mathcal{D}^\circ(U)_{\bar{c}, \dots, d, \gamma} = \{\underline{c} \in I : (\bar{c}, \underline{c}, d, \gamma) \in \mathcal{D}^\circ(U)\}$,

where $\mathcal{D}^\circ(U)_{\bar{c}, \dots, d, \gamma} = \begin{cases} [\alpha, \bar{c}) & \text{if } \bar{c} \in (\alpha, \beta], \\ \emptyset & \text{otherwise.} \end{cases}$

If $\underline{c} \in I$, then $\mathcal{D}^\circ(U)_{\dots, \underline{c}, d, \gamma} = \{\bar{c} \in I : (\bar{c}, \underline{c}, d, \gamma) \in \mathcal{D}^\circ(U)\}$,

where $\mathcal{D}^\circ(U)_{\dots, \underline{c}, d, \gamma} = \begin{cases} (\underline{c}, \beta] & \text{if } \underline{c} \in [\alpha, \beta), \\ \emptyset & \text{otherwise.} \end{cases}$

If $(\bar{c}, \underline{c}) \in (\alpha, \beta] \times [\alpha, \bar{c})$, then $\mathcal{D}^\circ(U)_{\bar{c}, \underline{c}, \dots, \gamma} = \{d \in \mathbb{R}_+^* : (\bar{c}, \underline{c}, d, \gamma) \in \mathcal{D}^\circ(U)\}$.

empirical concept of security (or safety) stock from the producer's point of view (see James H. Greene (1997)). Since we do not have any uncertainty on the demand side here, we will not use this term in this version of the paper.

4. To be clear we may use the following separable utility function : $U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma) = v(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) + \beta v(d_{t+1})$ (or $U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma) = v(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t, d_{t+1})$). Obviously, these utility function examples show that if $\gamma = 0$ then we have : $U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma) = v(\underline{c}_t) + \beta v(d_{t+1})$ ($U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma) = v(\underline{c}_t, d_{t+1})$ respectively).

Individuals are characterized by their separable inter-temporal utility function

$$U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma) = u(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) + \beta u(d_{t+1})$$

over the consumption set $\mathcal{D}^\circ(U)_{\bar{c},.,d,\gamma} \times \mathcal{D}^\circ(U)_{\bar{c},\underline{c},.,\gamma}$ during the two periods.

Assumption 12. The inter-temporal utility function U satisfies the following conditions :

1. The function U is of class \mathcal{C}^2 and strictly concave on $\mathcal{D}^\circ(U)$,
2. U is strictly increasing in both \underline{c}_t and d_{t+1} , or equivalently we have $\frac{\partial U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma)}{\partial \underline{c}_t} > 0$,
 $\frac{\partial U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma)}{\partial d_{t+1}} > 0$ on $\mathcal{D}^\circ(U)_{\bar{c},.,d,\gamma}$, and $\mathcal{D}^\circ(U)_{\bar{c},\underline{c},.,\gamma}$ respectively,

3. The Inada conditions are satisfied :

$$\lim_{\underline{c}_t \rightarrow 0} \frac{\partial U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma)}{\partial \underline{c}_t} = \infty, \quad \lim_{d_{t+1} \rightarrow 0} \frac{\partial U(\bar{c}_t, \underline{c}_t, d_{t+1}, \gamma)}{\partial d_{t+1}} = \infty,$$

for all $(\underline{c}_t, d_{t+1}) \in \mathcal{D}^\circ(U)_{\bar{c},.,d,\gamma} \times \mathcal{D}^\circ(U)_{\bar{c},\underline{c},.,\gamma}$.

B.2.1.1 Cash in Advance Constraint and Investment Abroad

Consider two countries. We suppose that capital is considered as being a quasi mobile factor of production. Indeed, individuals born at time t can invest their savings in the domestic as well as in the foreign capital market. They allocate their nominal wage W_t to the first period consumption \underline{c}_t , carry the nominal savings S_t and hold money balances M_t allowing them to buy goods in the second period of their lifetime. Given the total savings, consumers invest $S_t - \Sigma_t$ in their country and Σ_t in the foreign market capital. Note that the latter is allowed to move unilaterally across countries. However, unrestricted as well as restricted capital mobility are considered. We assume that each individual faces an exogenously given restriction, $\bar{\Sigma}$, on foreign investment. Thus the amount of capital an individual chooses to invest abroad must satisfy the following constraint $0 \leq \Sigma_t \leq \bar{\Sigma}$.

Their first period budget constraint is

$$p_t \underline{c}_t + (S_t - \Sigma_t) + \Sigma_t + M_t = W_t, \tag{B.1}$$

where p_t is the first period price of goods. Given the perfect anticipated price p_{t+1} , the perfect anticipated rates of return on capital R_{t+1} in country 1 and the perfect anticipated rate of return on capital invested abroad R_{t+1}^* , the savings realized in the first period allows each old individual to consume

$$p_{t+1}d_{t+1} = R_{t+1}(S_t - \Sigma_t) + R_{t+1}^*\Sigma_t + M_t, \quad (\text{B.2})$$

which is the second period budget constraint.

Given the consumer's income $W_t \in \mathbb{R}_+^*$ and the nominal rates of return $(R_{t+1}, R_{t+1}^*) \in (-1, \infty)^2$ on domestic and abroad savings respectively, in terms of real notations we denote by $s_t = \frac{S_t}{p_t}$, the real domestic savings, $\sigma_t = \frac{\Sigma_t}{p_t}$ the real abroad savings and $w_t = \frac{W_t}{p_t}$ the real wage earned by the young individuals in country 1. Under Assumption 3-1, rational individuals solve the following inter-temporal problem \mathcal{P} :

$$\mathcal{P} : \begin{cases} \text{Maximize} & u(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) + \beta u(d_{t+1}) \\ \text{w.r.t.} & (\underline{c}_t, d_{t+1}) \in \mathcal{D}^\circ(U)_{\bar{c}, \dots, d, \gamma} \times \mathcal{D}^\circ(U)_{\bar{c}, \underline{c}, \dots, \gamma} \\ \text{s.t.} & \underline{c}_t + s_t + \frac{M_t}{p_t} = w_t \\ & d_{t+1} = \frac{R_{t+1}}{p_{t+1}}(S_t - \Sigma_t) + \frac{R_{t+1}^*}{p_{t+1}}\Sigma_t + \frac{M_t}{p_{t+1}} \\ & \underline{c}_t \geq 0, d_{t+1} \geq 0. \end{cases} \quad (\text{B.3})$$

Inter-temporal budget constraint :

In terms of real notations, the second period budget constraint of individuals born in period t can be written as :

$$d_{t+1} = \frac{R_{t+1}p_t}{p_{t+1}}s_t + \left(\frac{R_{t+1}^*p_t}{p_{t+1}} - \frac{R_{t+1}p_t}{p_{t+1}} \right) \sigma_t + \frac{M_t}{p_{t+1}}, \quad (\text{B.4})$$

Isolating the savings function in both the first and the second period budget constrains, we obtain the following inter-temporal budget constraint of individuals in born t in country 1 :

$$\underline{c}_t + \frac{1}{\frac{R_{t+1}p_t}{p_{t+1}}}d_{t+1} + \frac{M_t}{p_t} \left(1 - \frac{1}{R_{t+1}} \right) + \left(1 - \frac{R_{t+1}^*}{R_{t+1}} \right) \sigma_t = w_t, \quad (\text{B.5})$$

The equation (B.5) can be rewritten as in Crettez, Michel and Wigniolle (1998, 1999) [27] to give the following real inter-temporal budget constraint allowing hereafter to solve the individuals problem :

$$\underline{c}_t + \frac{1}{R_{t+1}^{a,r}} d_{t+1} + \frac{M_t}{p_t R_{t+1}^{a,r}} \left(R_{t+1}^{a,r} - \frac{p_t}{p_{t+1}^a} \right) + \frac{\sigma_t}{R_{t+1}^{a,r}} \left(R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}^a} \right) = w_t, \quad (\text{B.6})$$

where the term $R_{t+1}^{a,r} = \frac{p_t R_{t+1}}{p_{t+1}}$ is the real anticipated rate of return on domestic savings and $\frac{p_t}{p_{t+1}^e}$ the real rate of return on the money holdings in country 1.

Assumption 13. Assuming that the capital is quasi-mobile across countries, we have

$$\frac{p_t}{p_{t+1}^a} < R_{t+1}^{a,r} < R_{t+1}^* \frac{p_t}{p_{t+1}^a}. \quad (\text{B.7})$$

The first inequality of (B.7) implies that individuals in country 1 hold the quantity of money imposed by the following constraint :

$$\frac{M_t}{p_{t+1}^a} = \mu d_{t+1}, \quad 0 < \mu < 1 \quad (\text{B.8})$$

where μ indicates the part of the second period consumption paid by the money held in the first period.

The second inequality of (B.7) indicates that individuals in country 1 invest abroad if the anticipated real rate of return on domestic savings is lower than the anticipated nominal interest rate on capital invested abroad weighted by the return on domestic money holding.

Note that the condition (B.8) means that the second period consumption is not totally financed by the money. This condition allows to ensure the capital accumulation in the economy.

B.2.1.2 Consumption and Savings

Using relation (B.8) of Assumption B.7 into (B.6) and rearranging, the inter-temporal constraint becomes :

$$\underline{c}_t + \left(\frac{1 - \mu}{R_{t+1}^{a,r}} + \mu \frac{p_{t+1}}{p_t} \right) d_{t+1} + \frac{\sigma_t}{R_{t+1}^{a,r}} \left(R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}^a} \right) = w_t, \quad (\text{B.9})$$

In the inter-temporal budget constraint, the second period consumption in country 1 is weighted by the sum of the inverse of the rates of return on domestic savings and money holding, while capital invested abroad is weighted by the difference between the real interest rates in the both countries. Replacing the second period consumption stemmed from the inter-temporal budget constraint (B.9) into the objective function, the inter-temporal problem \mathcal{P} becomes an unconstrained problem :

$$\begin{cases} \text{Maximize} & u(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) + \beta u(\vartheta_{t+1}(w_t - \underline{c}_t - \nu_{t+1}\sigma_t)) \\ \text{w.r.t.} & \underline{c}_t \in \mathcal{D}^\circ(U)_{\bar{c}, d, \gamma} \end{cases} \quad (\text{B.10})$$

where $\vartheta_{t+1} = \left(\frac{1-\mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-1}$ and $\nu_{t+1} = \frac{1}{R_{t+1}^r} (R_{t+1}^r - \frac{R_{t+1}^* p_t}{p_{t+1}})$ are respectively the factor of total rate of return on domestic savings and money holding and the factor of return on mobility of capital across countries. Given Assumption (12) the first order condition of the previous maximizing inter-temporal problem \mathcal{P} of individuals gives the following Euler equation :

$$u'(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) = \frac{\beta}{1-\gamma} \left(\frac{1-\mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-1} u'(d_{t+1}) \quad (\text{B.11})$$

The inter-temporal marginal rate of substitution (IMRS) which measures willingness to substitute consumption between the first period and the second period depends on the preference for quantity of individuals γ during the first period of their life-cycle.

The inter-temporal consumption behavior of individuals born in period t on the rate of time preference to preference for quantity relatively to combined rates of return on money holding and capital invested. The inter-temporal marginal rate of substitution is given by :

$$MRS_{\underline{c}_t, d_{t+1}} = \frac{u'(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t)}{\beta u'(d_{t+1})} \quad (\text{B.12})$$

Using (B.11) and (B.12), the marginal rate of substitution can be rewritten as :

$$MRS_{\underline{c}_t, d_{t+1}} = \frac{1}{(1 - \gamma) \left(\frac{1 - \mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)} \quad (\text{B.13})$$

Since the marginal utility is decreasing over the consumption set, the Euler equation (B.11) allows to distinguish different cases for a given parameter γ of preference for quantity.

$$\text{if } \beta \vartheta_{t+1} > 1 - \gamma, \quad u'_{\underline{c}_t} > u'_{d_{t+1}} \quad \Rightarrow \quad \underline{c}_t < d_{t+1}, \quad (\text{B.14})$$

$$\text{if } \beta \vartheta_{t+1} < 1 - \gamma, \quad u'_{\underline{c}_t} < u'_{d_{t+1}} \quad \Rightarrow \quad \underline{c}_t > d_{t+1}, \quad (\text{B.15})$$

$$\text{if } \beta \vartheta_{t+1} = 1 - \gamma, \quad u'_{\underline{c}_t} = u'_{d_{t+1}} \quad \Rightarrow \quad \underline{c}_t = d_{t+1}. \quad (\text{B.16})$$

The fact that the marginal rate of substitution adjusted by the preference for quantity is less than 1 means that individuals care a little more about current consumption than they care about future consumption. In the first case, individuals is inversely willing to allocate more important in favor of the second period consumption if the total rate of return on domesticating and money holding is higher than $1 - \gamma$.

Given the open set $\mathbb{A} = \mathcal{D}^\circ(U)_{., \underline{c}, d, \gamma} \times \mathcal{D}^\circ(U)_{\bar{c}, ., d, \gamma} \times \mathbb{R}_+^* \times \mathbb{R}_+^* \times \mathbb{R}_+^* \times]0, \bar{\Sigma}[\times]\underline{\gamma}, \bar{\gamma}[$, define a function f :

$$f : \mathbb{A} \rightarrow \mathbb{R} \\ (\bar{c}_t, \underline{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) \mapsto f(\bar{c}_t, \underline{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \mu, \sigma_t, \gamma)$$

such that

$$f(\bar{c}_t, \underline{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \gamma) :=$$

$$u'(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) - \frac{\beta}{1 - \gamma} \left(\frac{1 - \mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-1} u'(d_{t+1})$$

Consider $(\bar{c}_t^\circ, \underline{c}_t^\circ, w_t^\circ, R_{t+1}^\circ, R_{t+1}^{*\circ}, p_t^\circ, p_{t+1}^\circ, M_t^\circ, \sigma_t^\circ, \mu^\circ, \gamma^\circ) \in \mathbb{A}^\circ$ such that

$$f(\bar{c}_t^\circ, \underline{c}_t^\circ, w_t^\circ, R_{t+1}^\circ, R_{t+1}^{*\circ}, p_t^\circ, p_{t+1}^\circ, M_t^\circ, \sigma_t^\circ, \mu^\circ, \gamma^\circ) = 0.$$

Since by assumption 12.1 U is of class \mathcal{C}^2 , the function f is of class \mathcal{C}^1 on the open set \mathbb{A}° and $\frac{\partial^2 U}{\partial \underline{c}_t^2} < 0$, and $\frac{\partial^2 U}{\partial d_{t+1} c_t} > 0$, we have by (B.11) :

$$\frac{u''(\gamma(\bar{c}_t - \underline{c}_t))}{u''(d_{t+1})} \neq \frac{\beta}{(1-\gamma)^2} \left(\frac{1-\mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-2}. \quad (\text{B.17})$$

The latter implies that $\frac{\partial f}{\partial \underline{c}_t} \neq 0$. By the Implicit Function Theorem, there exists a neighborhood \mathcal{O}_1 and \mathcal{O}_2 of $(\bar{c}_t^\circ, w_t^\circ, R_{t+1}^\circ, R_{t+1}^{\star \circ}, p_t^\circ, p_{t+1}^\circ, M_t^\circ, \sigma_t^\circ, \mu^\circ, \gamma^\circ)$ and \underline{c}_t° respectively and a unique function g of class \mathcal{C}^1 such that :

$$\begin{aligned} g : \mathcal{O}_1 & \rightarrow \mathcal{O}_2 \\ (\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) & \mapsto g(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) \end{aligned}$$

Thus, for all $\underline{c}_t \in \mathcal{O}_2$, $(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, \sigma_t, \gamma) \in \mathcal{O}_1$,

we have : $f(\bar{c}_t, \underline{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) = 0$, if and only if :

$$\underline{c}_t = g(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma). \quad (\text{B.18})$$

Using the second period constraint into the first period constraint of \mathcal{P} , we have

$$s_t := s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) \quad (\text{B.19})$$

Using (B.18) into the first period constraint gives :

$$d_{t+1} = R_{t+1}^{a,r} s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) - R_{t+1}^{a,r} \nu_{t+1} \sigma_t + \frac{M_t}{p_{t+1}}. \quad (\text{B.20})$$

Note that if both the rates of interest of countries 1 and 2 are equal, then the second period consumption in country 1 depends only on the domestic savings and money holding. That is, the relation (B.20) becomes under Assumption (B.8) :

$$d_{t+1}^a = R_{t+1}^{a,r} s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) + \frac{M_t}{p_{t+1}}. \quad (\text{B.21})$$

B.2.1.3 The Demand for Money

Given the first inequality of (B.7) and the constraint imposed the cash in advance (B.8), the demand for money by young in period t is given by :

$$\frac{M_t}{p_t} = \mu \frac{p_{t+1}}{p_t} d_{t+1}^a, \quad 0 < \mu < 1. \quad (\text{B.22})$$

By replacing (B.20) into the (B.22), we have :

$$\frac{M_t}{p_t} = \mu \frac{p_{t+1}}{p_t} \left[R_{t+1}^{a,r} s_t + \frac{\sigma_t}{R_{t+1}^{a,r}} \left(R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}^a} \right) + \frac{M_t}{p_{t+1}} \right], \quad (\text{B.23})$$

By rearranging the previous expression, the demand for money is expressed as following :

$$\frac{M_t}{p_t} = \frac{\mu}{1 - \mu} \frac{p_{t+1}}{p_t} \left[R_{t+1}^{a,r} s_t + \frac{\sigma_t}{R_{t+1}^{a,r}} \left(R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}^a} \right) \right], \quad (\text{B.24})$$

Finally, since the savings depends on the demand for money in period t , which is denoted by $m_t = \frac{M_t}{p_t}$, with respect to the nominal rates of return on domestic savings and capital invested abroad, it is given implicitly by the following expression :

$$m_t = \frac{\mu}{1 - \mu} \left[R_{t+1}^a s_t + \frac{p_{t+1}}{p_t} \left(1 - \frac{R_{t+1}^*}{R_{t+1}^a} \right) \sigma_t \right]. \quad (\text{B.25})$$

The demand for real money obtained in (B.25), its derivative with respect to the domestic rate of return is obtained by :

$$\frac{\partial m_t}{\partial R_{t+1}^a} = \frac{\mu}{1 - \mu} \left[s_t + \frac{p_{t+1}}{p_t} \left(1 + \frac{R_{t+1}^*}{(R_{t+1}^a)^2} \right) \sigma_t \right] > 0. \quad (\text{B.26})$$

Note that the demand for money of young individuals depends on the anticipated rates of return on capital invested domestically and abroad. Given the perfect anticipated foreign interest rates R_{t+1}^* and price p_{t+1} , if the anticipated domestic interest rate $R_{t+1}^{a,r}$ increases, the demand for money is increasing. The latter allows individuals born in period t to consume more in the second period of their life-cycle.

Assumption 14. Given the anticipated rates on of return in both countries R_{t+1}, R_{t+1}^* , the anticipated price p_{t+1} and informations available in period $t : w_t, p_t, M_t, \sigma_t$, the real wage, the price of goods, the stock of money, the capital invested abroad respectively and the parameter μ and γ , the savings function s_t is of class \mathcal{C}^2 and increasing with respect to \bar{c}_t on the open set \mathcal{O}_1 , where $0 < s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma) < w_t$.

From the assumption of normality, we have

$$0 < \frac{\partial s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma)}{\partial w_t} < 1.$$

However, given the levels of the first and second period prices p_t and p_{t+1} , $\frac{\partial s}{\partial R_t}$ may be positive or negative, depending on substitution effect or revenue effect. Given p_t and p_{t+1} , if individuals are willing to substitute between the two periods consumption in order to take advantage on the rate of return incentives, the substitution effect dominates. In this case, the tradeoff between the first and second period consumption more favorable for the second period consumption tends to increase savings. On the contrary, the fact that a given amount of savings yields more second period consumption tends to decrease savings, the income effect dominates. In the last case, individuals have high preference for similar levels of consumption in the two periods.

When studying substitution and income effects after a change in rate of return on domestic savings, it is important to suppose that prices of the two periods consumption are fixed since these kinds of effects may be due to a change in prices p_t and p_{t+1} . In fact, an increase of second period price of consumption goods may allow consumer to consume less between the two periods of lifetime as long as individuals' purchasing power goes down as if their inter-temporal budget constraint changed or their income decreases. However, given the relative prices, individuals may switch between of the first and second period consumption of goods by keeping the level of utility unchanged. The particularity of substitution and income effects studied in this paper is function of the preference for quantity acted in the first period of individuals' lifetime.

Now we turn to study the production technology.

B.2.2 Production

Production occurs within a period according to a constant return to scale production technology which is stationary over time. The output Q_t of the single good produced at time t is given by the following neoclassical production function $Q_t = F(K_t, L_t)$ such that :

$$\begin{aligned} F : \mathbb{R}_+^* \times \mathbb{R}_+^* &\rightarrow \mathbb{R}_+^* \\ (K_t, L_t) &\mapsto F(K_t, L_t), \end{aligned}$$

Assumption 15. The production function F is of class \mathcal{C}^2 , increasing and concave. Thus we have :

$$\frac{\partial F(K_t, L_t)}{\partial K_t} > 0, \quad \frac{\partial F(K_t, L_t)}{\partial L_t} > 0, \quad \frac{\partial^2 F(K_t, L_t)}{\partial K_t^2} < 0, \quad \frac{\partial^2 F(K_t, L_t)}{\partial L_t^2} < 0, \quad \forall (K_t, L_t) \in \mathbb{R}_+^{*2}.$$

Assumption 16. The production function F is homogeneous of degree one (it reveals constant return to scale) and $F(K_t, 0) = F(0, L_t) = 0$.

Assumption 17. The production function F satisfies the Inada conditions :

$$\begin{aligned} \lim_{K_t \rightarrow 0} \frac{\partial F(K_t, L_t)}{\partial K_t} &= \lim_{K_t \rightarrow 0} \frac{\partial F(K_t, L_t)}{\partial L_t} = +\infty, \\ \lim_{K_t \rightarrow +\infty} \frac{\partial F(K_t, L_t)}{\partial K_t} &= \lim_{K_t \rightarrow +\infty} \frac{\partial F(K_t, L_t)}{\partial L_t} = 0. \end{aligned}$$

B.2.2.1 Capital Market γ -equilibrium

Denoting $k_t = \frac{K_t}{L_t}$ the capital per worker, given the real wage rate w_t and the domestic rate of return on capital R_t and foreign rate of return on capital R_t^* , the maximization profit problem of the firm that produces in period t is given by the following program :

$$\max_{k_t} p_t f(k_t) - W_t - R_t k_t, \tag{B.27}$$

Under Assumption (15), the first order condition leads to the following relations :

$$R_t = p_t f'(k_t) \tag{B.28}$$

$$w_t = f(k_t) - f'(k_t)k_t. \quad (\text{B.29})$$

There is an explicit link between the supply of capital and the demand for capital through σ_t^i . Indeed, taking into account the first and second period budget constraints, the demand for capital of the firm that produces is given by the following relation :

$$K_t = N_{t-1}(s_{t-1} - \sigma_{t-1}), \quad (\text{B.30})$$

Using the law of motion of the population that $N_t = (1 + n)N_{t-1}$, relation (B.30) can be written as follows :

$$(1 + n) \frac{K_t}{N_t} = s_{t-1} - \sigma_{t-1}. \quad (\text{B.31})$$

Since the capital per worker is not necessary equal to the capital per individual, we have the following expression :

$$(1 + n) \frac{L_t}{N_t} k_t = s_{t-1} - \sigma_{t-1},$$

which is expressed with respect to the domestic unemployment rate to give :

$$(1 + n)(1 - \theta_t)k_t = s_{t-1} - \sigma_{t-1}.$$

Finally, the capital demand per worker used in the production process is given by :

$$k_t = \frac{1}{(1 + n)(1 - \theta_t)} [s_{t-1} - \sigma_{t-1}], \quad (\text{B.32})$$

Since there exists a unique relation between the demand for capital per worker and the supply for capital per worker via the unemployment rate, given s_{t-1} which is chosen by individuals, the firm chooses alternatively the demand for capital per worker or the export of capital σ_t . Capital is considered as being a quasi mobile factor of production. Indeed, s_t always stays in the country 1, and solely σ_t is allowed to move across countries. The available quantity of capital is $N_{t-1}s_{t-1}$ and similarly, the available quantity of labor is L_t , so that involuntary unemployment is defined by $u_t = N_t - L_t$ while K_t satisfies the optimal conditions of the firm.

Given Assumption B.7 relative to the mobility of capital across countries and the domestic Cash-in-Advance constraint, by using relations (B.28) and (B.29), for all positive stock of capital available for production (Diamond, 1965), we have :

$$\frac{\partial w_t}{\partial k_t} = -f''(k_t)k_t. \quad (\text{B.33})$$

Using the equation (B.32), we have :

$$\frac{\partial w_t}{\partial k_t} = -\frac{f''(k_t)}{(1+n)} [s_{t-1} - \sigma_{t-1}]. \quad (\text{B.34})$$

Since the production function is concave by Assumption 15, the wage in period t is increasing with the demand for capital in country 1. However, the following derivative indicates that for a given price of the first period consumption of goods, the domestic rate of return on capital is a decreasing function of capital.

$$\frac{dR_t}{dk_t} = p_t f'' \left(\frac{1}{(1+n)} [s_{t-1} - \sigma_{t-1}] \right). \quad (\text{B.35})$$

For a given positive stock of capital, the relation (B.29) allows us to obtain the following variation of real wage with respect to the real rate of return on capital :

$$\frac{\partial w_t}{\partial R_t} = \frac{-1}{(1+n)} [s_{t-1} - \sigma_{t-1}] < 0. \quad (\text{B.36})$$

Using (B.28), the previous relation can be written as :

$$\frac{\partial w_t}{\partial R_t} = -f'^{-1} \left(\frac{R_t}{p_t} \right),$$

which implies

$$\frac{\partial^2 w_t}{\partial R_t^*} = \frac{-1}{p_t f'' \left(\frac{-1}{(1+n)} [s_{t-1} - \sigma_{t-1}] \right)} > 0.$$

Given the savings s_{t-1} realized by individuals born in period $t - 1$, the capital σ_{t-1} invested abroad in the previous period and the foreign rate of return capital R_{t+1}^* , the latter implies that the wage is a decreasing and convex function in the rate of return.

We now turn to the study of the National Accounting.

B.2.3 National Accounting

Before determining the temporary γ -equilibrium of the open economy, we set up the national accounting of each country. Note that the Walras' law does not apply here. There are four markets. The goods market, the labor market, the monetary market and the capital market. There are four kinds of prices but only two are the same, the price of the current saving or equivalently the real price of domestic money. The price of domestic produced goods is p_t , the price of labor or the real wage is W_t and the prices of capital is (R_t, R_t^*) . We suppose that there are no flows of goods, services, primary income, and secondary income between residents and non-residents of two countries.

Assuming that capital is a quasi mobile across countries, while labor is momentarily an already fixed factor of production.

Agents : Nominal Uses = Nominal Resources

1. Young : $N_t p_t \bar{c}_t + N_t S_t + N_t M_t = N_t W_t$
2. Old : $N_{t-1} p_t d_t = R_t N_{t-1} (S_{t-1} - \Sigma_{t-1}) + R_t^* N_{t-1} \Sigma_{t-1} + N_{t-1} M_{t-1}$
3. Firms : $N_t p_t (\bar{c}_t - c_t) + W_t L_t + R_t N_{t-1} (S_{t-1} - \Sigma_{t-1}) = p_t Q_t$
4. Inv. : $I_t = N_t (S_t - \Sigma_t)$
5. BoK $N_t \Sigma_t = R_t^* N_{t-1} \Sigma_{t-1} + D_f$

Aggregating all uses and resources provides us with the following expressions.

$$\begin{cases} N_t p_t \bar{c}_t + N_{t-1} d_t + I_t + N_t \Sigma_t + N_t M_t - N_{t-1} M_{t-1} = \\ p_t Q_t + W_t (N_t - L_t) + R_t^* N_{t-1} \Sigma_{t-1} + \\ N_t \Sigma_t = R_t^* N_{t-1} \Sigma_{t-1} + D_f \end{cases}$$

where D_f is the deficit/excess of the balance of capital, $N_t M_t$ is the demand for money.

Assumption 18. We suppose that there is no creation or destruction of money by the government after $t = 0$. The supply of money \bar{M} is inelastic and equal to stock of money $N_{t-1}M_{t-1}$ held by the old individual.

In terms of real and intensive variables, the previous aggregate national account can be rewritten as the following :

$$p_t \bar{c}_t + \frac{1}{1+n} d_t + \frac{I_t}{N_t p_t} + \sigma_t + \frac{1}{p_t} (M_t - \frac{M_{t-1}}{1+n}) = \frac{L_t}{N_t} q_t + w_t (\frac{N_t - L_t}{N_t}) + \frac{R_t^*}{1+n} \sigma_{t-1} \frac{p_{t-1}}{p_t}.$$

Denoting by $\theta_t = \frac{N_t - L_t}{N_t}$ the domestic unemployment rate, the notional account of the open economy is given by the following :

$$\bar{c}_t + \frac{1}{1+n} d_t + \frac{I_t}{N_t p_t} + \sigma_t + m_t - \frac{m_{t-1}}{1+n} = (1 - \theta_t) q_t + \theta_t w_t + \frac{R_t^*}{1+n} \sigma_{t-1} \frac{p_{t-1}}{p_t}, \quad (\text{B.37})$$

where m_t is the real money held in the period t for consumption in the period $t + 1$.

Remark 2. In an economy with n markets, the Walras' Law specifies that if $n - 1$ markets are in equilibrium (there exists a price that clears the market), then necessarily the n th market is in equilibrium. In our case, since at the market γ -equilibrium price $p_t^*(\gamma)$ the goods market does not clear, we have the following statement, concerning the contrapositive of the Walras' Law :

If one market is in γ -equilibrium, then there exists at least another one that is in γ -equilibrium.

Since the goods market is in γ -equilibrium, one at least of the labor market, the money market or the capital market is in γ -equilibrium. We suppose that under the contrapositive Walras' Law both the labor market and monetary market clear and the capital market is in γ -equilibrium.

Given the remark 2, we have :

$$\bar{c}_t + \frac{1}{1+n}d_t + \frac{I_t}{N_t p_t} + \sigma_t = q_t + \frac{R_t^*}{1+n}\sigma_{t-1}\frac{p_{t-1}}{p_t}. \quad (\text{B.38})$$

The macroeconomic equilibrium (B.38) expresses that the global supply equals the global demand, where the right-hand side represents the sum of the production and the return capital invested abroad in period $t - 1$. The global supply available in period t is allocated to the quantity of displayed goods for young, the consumption by old individuals, the real investment per young and the stock of capital invested abroad in period t .

It results that the stock of unsold goods $\Delta_t = \bar{c}_t - \underline{c}_t$ is obtained by the national accounting :

$$\Delta_t = q_t - \left[\underline{c}_t + \frac{1}{1+n}d_t + \frac{I_t}{N_t p_t} + \sigma_t - \frac{R_t^*}{1+n}\sigma_{t-1}\frac{p_{t-1}}{p_t} \right] \quad (\text{B.39})$$

If both the stock of unsold goods Δ_t and σ_t vanish the relation (B.39) of the national accounting is equivalent to the result obtained in Crettez et al.(1998).

B.3 Autarky

Under Autarky, individuals born in period t invest their capital in their country since there is neither inflow nor outflow of capital across countries. The capital dynamics in country 1 depends on the young decision in period t . With the Log utility function and Cobb-Douglas production function, the savings function is increasing in quantity of good \bar{c}_t . Given anticipated rate of return of capital R_{t+1} , since the quantity of displayed goods increases in preference for quantity γ , the stock of capital must increase in order for producing firm to satisfy production of goods.

B.3.1 Autarkic Inter-temporal γ -equilibrium

Note that all the preceding relations remain true except that we assume no mobility of across countries, that is : $\bar{\Sigma} = 0$. Under Autarky, the national accounting (B.38) becomes :

$$\Delta_t + \underline{c}_t + \frac{1}{1+n}d_t + \frac{1}{p_t} \frac{I_t}{N_t} = q_t, \quad (\text{B.40})$$

Definition 7. An inter-temporal autarkic γ -equilibrium is defined as three sequences : the sequence of prices $(w_t, R_t, p_t)_{t \in \mathbb{N}}$, the sequence of individual quantities $(\bar{c}_t, \underline{c}_t, d_t, s_t)_{t \in \mathbb{N}}$, and the sequence of aggregate quantities $(K_t, N_t, Q_t)_{t \in \mathbb{N}}$ which satisfies (B.3), (B.6), (B.7), (B.8) and (B.40), (the optimality condition of households), and the following conditions :

$$w_t = f(k_t) - f'(k_t)k_t \quad (\text{B.41})$$

$$R_t = p_t f'(k_t) \quad (\text{B.42})$$

$$k_{t+1} = \frac{1}{1+n} s(\bar{c}_t, p_t(f(k_t) - f'(k_t)k_t), p_{t+1}f'(k_{t+1}), M_t, p_{t+1}, p_t, \mu, \gamma) \quad (\text{B.43})$$

The existence of inter-temporal autarkic γ -equilibrium is ensured by Assumption (14), the uniqueness of the firm maximization problem (B.28 and B.29) and (B.40). Given μ, γ , there exists $k > 0$ such that :

$$0 < w(k_t) - s(\bar{c}_t, w_t, R_{t+1}, R_{t+1}^*, p_t, p_{t+1}, M_t, \sigma_t, \mu, \gamma). \quad (\text{B.44})$$

The previous condition means that whatever the level of anticipated rate of return on capital, the savings in period t does not exceed the real wage for any positive stock of capital in country 1.

Definition 8. Given the preceding remark, the steady-state γ -equilibrium is a stationary sequence of the price vector $(\hat{W}, \hat{R}, \hat{p})$ verifies the following conditions :

$$\hat{W} = \hat{p}(f(k) - f'(k)k)$$

$$\hat{R} = \hat{p}f'(k)$$

$$Q = Nf(k)$$

$$k = \frac{1}{(1+n)} S(\bar{c}, \hat{W}, \hat{R}, M, \hat{p}, \mu, \gamma)$$

$$I = NS$$

$$\hat{p}\underline{c} = \hat{W} - S - M$$

$$\frac{M}{\hat{p}} = \frac{\mu}{1-\mu} \hat{R}s$$

$$\hat{p}d = \hat{R}S + M$$

$$\hat{p}\Delta = q - (\underline{c} + \frac{d}{1+n} + S) \geq 0.$$

Lemma 6. The autarkic steady-state γ -equilibrium with existence of money is locally stable if the following condition is satisfied :

$$0 < 1 + n + pf''(k) \left[k \frac{\partial S}{\partial w} - \frac{\partial S}{\partial R} \right] - \frac{\partial \bar{c}}{\partial k} \frac{\partial S}{\partial \bar{c}} - \frac{\partial M}{\partial k} \frac{\partial S}{\partial M}.$$

If $\gamma = 0$, then the local stability of steady-state is given by the following condition :

$$0 < 1 + n + pf''(k) \left[k \frac{\partial S}{\partial w} - \frac{\partial S}{\partial R} \right] - \frac{\partial M}{\partial k} \frac{\partial S}{\partial M}.$$

Proof. Using the relation of the dynamics of capital (B.43), we have :

$$(1+n) \frac{dk_{t+1}}{dk_t} = \frac{\partial \bar{c}}{\partial k_t} \frac{\partial S_t}{\partial \bar{c}_t} - p_t f''(k_t) k_t \frac{\partial S_t}{\partial w_t} - p_{t+1} \frac{dk_{t+1}}{dk_t} f''(k_{t+1}) \frac{\partial S_t}{\partial R_{t+1}} + \frac{\partial M_t}{\partial k_t} \frac{\partial S_t}{\partial M_t}$$

Regroup terms with respect to dk_{t+1}/dk_t , and obtain

$$\left[1 + n - p_{t+1} f''(k_{t+1}) \frac{\partial S}{\partial R_{t+1}} \right] \frac{dk_{t+1}}{dk_t} = \frac{\partial \bar{c}}{\partial k_t} \frac{\partial S}{\partial \bar{c}} + \frac{\partial M_t}{\partial k_t} \frac{\partial S}{\partial M_t} - p_t f''(k_t) k_t \frac{\partial S}{\partial w_t},$$

that implies

$$\frac{dk_{t+1}}{dk_t} = \frac{\frac{\partial \bar{c}}{\partial k_t} \frac{\partial S}{\partial \bar{c}} + \frac{\partial M_t}{\partial k_t} \frac{\partial S}{\partial M_t} - p_t f''(k_t) k_t \frac{\partial S}{\partial w_t}}{1 + n - p_{t+1} f''(k_{t+1}) \frac{\partial S}{\partial R_{t+1}}}.$$

There exists a local stability is the following condition holds :

$$\left| \frac{dk_{t+1}}{dk_t} \right|_{k_t=\hat{k}} < 1,$$

which is equivalent to :

$$\frac{\partial \bar{c}}{\partial k} \frac{\partial S}{\partial \bar{c}} + \frac{\partial M}{\partial k} \frac{\partial S}{\partial M} - pf''(k)k \frac{\partial S}{\partial w_t} < 1 + n - pf''(k) \frac{\partial S}{\partial R}$$

which completes the proof of the first part of Lemma (6).

To proof the second part the lemma, it is sufficient to note that if $\gamma = 0$ the young individuals do not have any preference stock of unsold goods, that is : $\gamma(\bar{c}_t - c_t) + c_t = c_t$. Thus the savings function is independent of \bar{c}_t , since the inter-temporal utility function is independent of \bar{c}_t .

□

Note that, in the case of $\gamma > 0$, i.e the principle of preference for quantity is active, the local stability of the steady-state γ -equilibrium depends on the condition of the national account, where Δ stands for the stock of unsold goods remaining at the end of the period t . However, if the preference for quantity does not hold, then the stock of unsold goods Δ is equal to zero, that is case of the second part of Lemma (6).

B.4 International Capital Mobility

In this section, we suppose that factor of capital mobility does not vanish, that is : $\bar{\Sigma} \neq 0$. Given the contrapositive of Walras' Law, we assume that all individuals born in period t are hired, that is :

$$\theta_t = \frac{N_t - L_t}{N_t} = 0.$$

Thus, the temporary γ -equilibrium is defined as follows.

Definition 9. Given the previous period variables s_{t-1} , $I_{t-1} = N_{t-1}s_{t-1}$ and the perfectly anticipated rate of return on capital R_{t+1} , a temporary γ -equilibrium with unrestricted capital mobility is defined by the following expressions :

$$w_t = f(k_t) - f'(k_t)(k_t)$$

$$R_t = p_t f'(k_t)$$

$$Q_t = N_t f(k_t)$$

$$k_t = \frac{1}{(1+n)} [s_{t-1} - \sigma_{t-1}]$$

$$I_t = N_t (s_t - \Sigma_t)$$

$$c_t = w_t - s_t - m_t$$

$$m_t = \frac{\mu}{1-\mu} \left[R_t^a s_{t-1} + \frac{p_t}{p_{t-1}} \left(1 - \frac{R_t^*}{R_t} \right) \sigma_{t-1} \right]$$

$$d_t = R_t^r s_t + \frac{\sigma_{t-1}}{R_t^r} \left(R_t^r - R_t^* \frac{p_{t-1}}{p_t} \right) + \frac{p_{t-1}}{p_t} m_{t-1}.$$

B.4.1 The role of inflation on the stock of unsold goods at the γ -equilibrium

Considering the first period constraint of individuals born in period t , the first period consumption can be written as :

$$\underline{c}_t = w_t - s_t - \frac{M_t}{p_t}$$

Given the solution of the maximization problem of the producing firm and the dynamics of capital, the first period consumption is expressed by :

$$\underline{c}_t = f(k_t) - R_t k_t - (1+n)k_{t+1} - \sigma_t - \frac{M_t}{p_t} \quad (\text{B.45})$$

By the liquidity constraint, the second period consumption is obtained by replacing the savings from the capital dynamics :

$$d_t = \frac{R_t p_{t-1}}{p_t} [(1+n)k_t + \sigma_{t-1}] + \left(\frac{R_t^* p_{t-1}}{p_t} - \frac{R_t p_{t-1}}{p_t} \right) \sigma_{t-1} + \frac{M_{t-1}}{p_t}, \quad (\text{B.46})$$

$$d_t = \frac{p_{t-1}}{p_t} \left[(1+n)R_t k_t + R_t^* \sigma_{t-1} + \frac{M_{t-1}}{p_{t-1}} \right], \quad (\text{B.47})$$

Given the cash in advance constraint, we have :

$$d_t = \frac{1}{1-\mu} \frac{p_{t-1}}{p_t} [(1+n)R_t k_t + R_t^* \sigma_{t-1}]. \quad (\text{B.48})$$

The derivatives of γ -equilibrium consumption with respect to the domestic and foreign rates of return capital are given by :

$$\frac{\partial d_t}{\partial R_t} = \frac{1}{1-\mu} \frac{p_{t-1}}{p_t} (1+n)k_t, \quad (\text{B.49})$$

$$\frac{\partial d_t}{\partial R_t^*} = \frac{1}{1 - \mu} \frac{p_{t-1}}{p_t} \sigma_{t-1}. \quad (\text{B.50})$$

The relation (B.48) exhibits the influence of inflation on the consumption decision. It is obvious that the increase of rate of return R_t on domestic capital allows the old to consume more depending on the level of inflation rate in the economy.

Lemma 7. Given the principle of preference for quantity, the existence of stock of unsold goods at the γ -equilibrium generates inflation and the excess of money supply in the economy.

Proof. Show that the stock of unsold goods is a function of inflation and the money demand. Since the rate of return on capital invested abroad is higher than that of domestic capital, we can assume that the capital imported vanishes. Thus, the relation (B.39) of stock of γ -equilibrium becomes :

$$\Delta_t = f(k_t) - \left[c_t + \frac{1}{1+n} d_t + \frac{I_t}{N_t p_t} + \sigma_t - \frac{R_t^*}{1+n} \sigma_{t-1} \frac{p_{t-1}}{p_t} \right] \quad (\text{B.51})$$

Note that equality between the nominal uses and resources of the producing firm implies :

$$I_t = N_t(S_t - \Sigma_t)$$

Using the equation of capital dynamics, the previous identity can be rewritten in real terms as follows :

$$\frac{I_t}{p_t N_t} = (1+n)k_{t+1}. \quad (\text{B.52})$$

Replacing (B.45), (B.48) and (B.52) into (B.51) and simplifying, we have :

$$\Delta_t = \left(1 - \frac{p_{t-1}}{p_t}\right) R_t k_t + \frac{1}{p_t} \left(M_t - \frac{M_{t-1}}{1+n}\right) \quad (\text{B.53})$$

Since the stock of capital is positive, for the stock of unsold goods to be non negative, we have the following conditions :

$$p_t > p_{t-1} \quad \text{and} \quad M_t < \frac{M_{t-1}}{1+n} \quad (\text{B.54})$$

$$p_t < p_{t-1} \quad \text{and} \quad M_t > \frac{M_{t-1}}{1+n} \quad (\text{B.55})$$

$$p_t = p_{t-1} \quad \text{and} \quad M_t > \frac{M_{t-1}}{1+n} \quad (\text{B.56})$$

$$p_t > p_{t-1} \quad \text{and} \quad M_t = \frac{M_{t-1}}{1+n} \quad (\text{B.57})$$

Denoting the inflation rate by :

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$$

The case (B.54) indicates that the existence of stock of unsold goods may generate inflation and the excess of money supply in the economy. If the demand for money matches the money supply, the stock of unsold goods depend on the capital of γ -equilibrium and the level of inflation rate π_t . Given the stock of capital and the rate of return, if inflation increases then the stock of unsold goods increases.

□

Remark 3. Note that the steady-state stock of unsold goods depends only on the steady-state stock of money held by individuals and the rate of domestic population growth. Indeed, at the steady-state the relation (B.53) of stock of unsold goods becomes :

$$\Delta = \frac{n}{1+n} m, \quad (\text{B.58})$$

where m is the real money held by individuals. Given the expression of real money, we have :

$$\Delta = \frac{n}{1+n} \frac{\mu}{1+\mu} \left[R_s + \left(1 - \frac{R^*}{R}\right) \sigma \right] \quad (\text{B.59})$$

The previous expression indicates that at the steady-state γ -equilibrium, the stock of unsold goods is a positive function of the capital invested abroad.

B.4.2 Steady-state Welfare Implications of capital with Stock of Unsold Goods

Consider the following inter-temporal indirect utility function of individuals born in period t :

$$v_t = v(\bar{c}_t, w_t, p_t, R_{t+1}, R_{t+1}^*, p_{t+1}, \mu, \gamma) \quad (\text{B.60})$$

The optimal utility of each individual born in period t depends on the perfect-foresight return on his capital invested and price of his second period consumption of goods, the money holding and his preference for quantity during his first period of lifetime. Since the wage and return on capital invested depends on the capital by worker in country 1, we have the following theorem concerning the steady-state welfare.

Theorem 4. Given the part of the second period consumption paid by the money held in the first period μ , given the prices p_t and p_{t+1} of the first and second period consumption \bar{c}_t and d_{t+1} respectively, and the anticipated rate of return on capital invested abroad R_{t+1}^* , the steady-state welfare without as well as with stock of unsold goods is an increasing function of domestic demand for capital if the following condition holds :

$$\frac{\frac{\partial v}{\partial R}}{\frac{\partial v}{\partial w}} < \frac{1}{p} \frac{\partial w}{\partial R}, \quad \text{with} \quad \frac{\partial w}{\partial R} < 0, \text{ if } \gamma = 0, \quad (\text{B.61})$$

$$\left[\frac{1}{1+n} (s - \sigma) \frac{\partial v}{\partial w} - p \frac{\partial v}{\partial R} \right] f''(k) < \frac{\partial \bar{c}}{\partial k} \frac{\partial v}{\partial \bar{c}}, \quad \text{if } \gamma > 0. \quad (\text{B.62})$$

Proof. If $\gamma = 0$ then the inter-temporal utility function of individuals born in period t is independent of the demand for \bar{c}_t . Thus there is no need to take into account the stock unsold goods in the problem \mathcal{P} . By taking the derivative of indirect utility function (B.60) with respect to the stock of capital per worker, we have :

$$\frac{\partial v_t}{\partial k_t} = \frac{\partial w_t}{\partial k_t} \frac{\partial v_t}{\partial w_t} + \frac{\partial R_{t+1}}{\partial k_t} \frac{\partial v}{\partial R}. \quad (\text{B.63})$$

Using (B.34) and the fact that $\frac{\partial R_{t+1}}{\partial k_t} = p_{t+1} \frac{\partial k_{t+1}}{\partial k_t} f''(k_{t+1})$, the previous relation can be written as follows :

$$\frac{\partial v_t}{\partial k_t} = -\frac{f''(k_t)}{(1+n)} [s_{t-1} - \sigma_{t-1}] \frac{\partial v_t}{\partial w_t} + p_{t+1} \frac{\partial k_{t+1}}{\partial k_t} f''(k_{t+1}) \frac{\partial v}{\partial R}. \quad (\text{B.64})$$

At the steady-state γ -equilibrium, the latter can be expressed as :

$$\frac{\partial \tilde{v}}{\partial \tilde{k}} = f''(\tilde{k}) \left[\frac{-1}{(1+n)} (s - \sigma) \frac{\partial \tilde{v}}{\partial \tilde{w}} + \tilde{p} \frac{\partial \tilde{v}}{\partial \tilde{R}} \right]. \quad (\text{B.65})$$

Since by Assumption 15 $f''(\tilde{k}) < 0$, we have :

$$\text{sgn} \left[\frac{\partial \tilde{v}}{\partial \tilde{k}} \right] = \text{sgn} \left[\frac{-1}{(1+n)} (s - \sigma) \frac{\partial \tilde{v}}{\partial \tilde{w}} + \tilde{p} \frac{\partial \tilde{v}}{\partial \tilde{R}} \right],$$

from which we deduce that

$$\frac{\partial \tilde{v}}{\partial \tilde{k}} \geq 0 \quad \text{if and only if} \quad \frac{\frac{\partial \tilde{v}}{\partial \tilde{R}}}{\frac{\partial \tilde{v}}{\partial \tilde{w}}} \leq \frac{1}{p_t} \frac{\partial \tilde{w}}{\partial \tilde{R}},$$

where $\frac{\partial w}{\partial R} < 0$ according to relation (B.36).

If $\gamma > 0$, then the inter-temporal indirect utility function (B.60) allows the following derivative with respect to k :

$$\frac{\partial v_t}{\partial k_t} = \frac{\partial \bar{c}}{\partial k} \frac{\partial v}{\partial \bar{c}} - f''(k_t) k \frac{\partial v_t}{\partial w_t} + p_{t+1} \frac{\partial k_{t+1}}{\partial k_t} f''(k_{t+1}) \frac{\partial v}{\partial R}. \quad (\text{B.66})$$

Using the same procedure as we did to show (B.62), the derivative $\frac{\partial v_t}{\partial k_t}$ is positive in the neighborhood of the steady-state γ -equilibrium if we have :

$$\frac{\partial \bar{c}}{\partial k} \frac{\partial v}{\partial \bar{c}} > \left[\frac{1}{1+n} (s - \sigma) \frac{\partial v}{\partial w} - p \frac{\partial v}{\partial R} \right] f''(k),$$

where k is given by the equation (B.32).

□

B.4.3 Steady-state welfare implications of capital mobility with stock of unsold goods and money

The theorem 4 gives the condition for which the steady-state welfare is an increasing function of capital in country 1 for a given level of preference for quantity γ . However, the following proposition exhibits the steady-state welfare implications of capital mobility with the presence of preference for quantity, consequently the stock of unsold goods such that $\Delta_t \neq 0$.

Proposition 5. Given the world steady-state γ -equilibrium, the steady-state welfare of individuals born in country 1 is an increasing function of capital mobility

Proof. Given the objective function of unconstrained problem (B.10), the first period constraint of constrained problem, the inter-temporal indirect utility function (B.60) can be rewritten as follows :

$$v_t = u(\gamma\bar{c}_t + (1 - \gamma)(w(k_k) - s(\sigma_t) - m(\sigma_t))) + \beta u(\vartheta_{t+1}(s(\sigma_t) + m(\sigma_t) - \nu_{t+1}\sigma_t)) \quad (\text{B.67})$$

Replacing the demand for money into (B.67), which is differentiated with respect to σ_t to give :

$$\frac{dv_t}{d\sigma_t} = \eta_t u'_c + \beta \vartheta_{t+1} \eta_{t+1} u'_{d_{t+1}}, \quad (\text{B.68})$$

where

$$\eta_{1t} = \gamma \frac{d\bar{c}_t}{d\sigma_t} + (1 - \gamma) \left(\frac{\partial w_t}{\partial \sigma_t} - \left(1 + \frac{\mu}{1 - \mu} R_{t+1}\right) \frac{\partial s(\sigma_t)}{\partial \sigma_t} - \frac{\mu}{1 - \mu} \frac{p_{t+1}}{p_t} \left(1 - \frac{R_{t+1}^*}{R_{t+1}}\right) \right) \quad (\text{B.69})$$

and

$$\eta_{2t+1} = \beta \vartheta_{t+1} \left(\left(1 + \frac{\mu}{1 - \mu} R_{t+1}\right) \frac{\partial s(\sigma_t)}{\partial \sigma_t} - \left(1 - \frac{\mu}{1 - \mu} \frac{p_{t+1}}{p_t}\right) \left(1 - \frac{R_{t+1}^*}{R_{t+1}}\right) \right) \quad (\text{B.70})$$

At the steady-state γ -equilibrium the derivative of the indirect utility function with respect to the stock of capital invested abroad can be written as :

$$\frac{dv}{d\sigma} = \eta_1 u'_c + \beta \vartheta \eta_2 u'_d, \quad (\text{B.71})$$

By assumption, u'_c and u'_d are non negative and the total rate of return on saving invested abroad and money holding is positive, the sign of (B.71) depends on the following steady-state factors :

$$\eta_1 = \gamma \frac{d\bar{c}}{d\sigma} + (1 - \gamma) \left(\frac{\partial w}{\partial \sigma} - \left(1 + \frac{\mu}{1 - \mu} R\right) \frac{\partial s(\sigma)}{\partial \sigma} - \frac{\mu}{1 - \mu} \left(1 - \frac{R^*}{R}\right) \right)$$

and

$$\eta_2 = \beta \vartheta \left(\left(1 + \frac{\mu}{1 - \mu} R\right) \frac{\partial s(\sigma)}{\partial \sigma} - \left(1 - \frac{\mu}{1 - \mu}\right) \left(1 - \frac{R^*}{R}\right) \right)$$

□

B.4.4 Exemple : CES-utility and Cobb-Douglas production functions

Consider a two-country overlapping generations model with constant inter-temporal elasticity of substitution (CIES) utility function and a Cobb-Douglas production. Hence each individual born in period t in country 1 is characterized by the following inter-temporal CES-utility function :

$$U(\bar{c}_t, \underline{c}_t, d_{t+1}) = a [\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t]^\rho + (1 - a) d_{t+1}^\rho \quad (\text{B.72})$$

where γ is the parameter of the preference for quantity, a the share parameter and $\frac{1}{\rho}$ the elasticity of substitution. Each individual maximizes his function with respect to the cash in advance constraint and both the first and second period constraints. The first period consumption is given by :

$$\underline{c}_t = \frac{\left[\frac{(1-a)}{a} \frac{(\vartheta_{t+1}^\rho)}{(1-\gamma)} \right]^{\frac{1}{\rho-1}} [w_t - \nu_{t+1} \sigma_t] - \gamma \bar{c}_t}{1 - \gamma + \left[\frac{(1-a)}{a} \frac{(\vartheta_{t+1}^\rho)}{(1-\gamma)} \right]^{\frac{1}{\rho-1}}} \quad (\text{B.73})$$

or by simplifying, we have :

$$\underline{c}_t = \frac{[w_t - \nu_{t+1}\sigma_t] - \gamma \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{1-\rho}} \bar{c}_t}{1 + (1-\gamma) \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{1-\rho}}} \quad (\text{B.74})$$

Since the total factor of anticipated rate of return ϑ_{t+1} on money and capital is positive, the first period consumption is decreasing in the quantity of production \bar{c}_t . Note that the latter plays a role of parameter in individuals maximization problem. Furthermore, the positivity condition of consumption \underline{c}_t implies :

$$\bar{c}_t \leq \frac{[w_t - \nu_{t+1}\sigma_t]}{\gamma \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{1-\rho}}} \quad (\text{B.75})$$

where by definition $\vartheta_{t+1} > 0$. The savings function is obtained by replacing (B.75) into the first period constraint (B.88) of individual :

$$s_t = \frac{(1-\gamma)w_t + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}} \nu_{t+1}\sigma_t + \gamma\bar{c}_t}{1 - \gamma + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}}} - \frac{M_t}{p_t} \quad (\text{B.76})$$

The second period consumption is given by :

$$d_{t+1} = R_{t+1}^r \left[\frac{(1-\gamma)w_t + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}} \nu_{t+1}\sigma_t + \gamma\bar{c}_t}{1 - \gamma + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}}} \right] - \left(\frac{M_t}{p_t} - R_{t+1}^r \right) m_t + \left(\frac{R_{t+1}^* - R_{t+1}}{p_{t+1}} \right) \sigma_t \quad (\text{B.77})$$

It results from the second period consumption and the constraint imposed by the cash in advance constraint (B.8) that the demand for money in period t is given by :

$$m_t = \zeta_t \left[R_{t+1} \frac{(1-\gamma)w_t + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}} \nu_{t+1}\sigma_t + \gamma\bar{c}_t}{1 - \gamma + \left[\frac{(1-a)}{a} \frac{\vartheta_{t+1}^\rho}{(1-\gamma)} \right]^{\frac{1}{\rho-1}}} + p_{t+1} \left(\frac{R_{t+1}^* p_t}{p_{t+1}} - R_{t+1}^r \right) \sigma_t \right] \quad (\text{B.78})$$

with $\zeta_t = \frac{\mu}{1 + \mu(R_{t+1} - 1)}$.

B.4.5 Optimal Stationary Paths with Stock of unsold Goods

This subsection hinges on optimal stationary paths with existence of stock of unsold goods and liquidity constraint of individuals born in period t in country 1. Given the national accounting identity (B.39), given the previous variables s_{t-1} , σ_{t-1} , $I_{t-1} = N_{t-1}s_{t-1}$ and suppose that both the parameter of preference for quantity is active at the first period of individuals' life-cycle and existence of mobility of capital unilaterally, the intertemporal γ -equilibrium is defined by :

$$f(k_t) = \underline{c}_t + \frac{1}{1+n}d_t + \frac{1+n}{p_t}k_{t+1} + \sigma_t - \frac{R_t^*}{1+n} \frac{p_{t-1}}{p_t} \sigma_{t-1} + \Delta_t \quad (\text{B.79})$$

As in De la Croix and Michel (2002), if the steady-state stock of capital \tilde{k} is strictly positive, the first and second period consumption, the total savings and the stock of money converge to :

$$\begin{aligned} \tilde{c} &= w(\tilde{k}) - \tilde{s} - \tilde{m} \\ \tilde{d} &= R\tilde{s} + \tilde{\sigma}(1 - \frac{R^*}{R}) + \tilde{m} \\ \tilde{s} &= s(w(\tilde{k}), f'(\tilde{k}), \tilde{\sigma}) \end{aligned}$$

Furthermore, the stock of capital \tilde{k} verifies also the following resource constraint :

$$f(\tilde{k}) = \tilde{c} + \frac{1}{1+n}\tilde{d} + \frac{1+n}{p}\tilde{k} + (1 - \frac{R^*}{1+n})\tilde{\sigma} + \tilde{\Delta}$$

Assuming that the parameter of preference for quantity $\gamma > 0$, the highest stationary utility of individuals born in period t is given by the following problem :

$$\begin{cases} \text{Maximize } u(\gamma\bar{c} + (1-\gamma)\underline{c}) + \beta u(\vartheta(w - \underline{c} - \nu\sigma)) \\ \text{s.t. } f(k) = \underline{c} + \frac{1}{1+n}d + \frac{1+n}{p}k + (1 - \frac{R^*}{1+n})\sigma + \Delta \end{cases} \quad (\text{B.80})$$

Denoting by $\tilde{\phi}(k)$ the net domestic production, we have :

$$\tilde{\phi}(k) = f(k) - \frac{1+n}{p}k - (1 - \frac{R^*}{1+n})\sigma. \quad (\text{B.81})$$

Thus the domestic resource constraint becomes :

$$\tilde{\phi}(k) = \underline{c} + \frac{1}{1+n}d + \Delta \quad (\text{B.82})$$

Recall that at the steady-state γ -equilibrium the stock of unsold goods is given by the relation (B.58). The relation (B.82) can be written as :

$$\tilde{\phi}(k) = \underline{c} + \frac{1}{1+n}d + \frac{n}{1+n}m \quad (\text{B.83})$$

where $\underline{c} \geq 0$, $d \geq 0$, $m \geq 0$.

Given above, the net production is positive, i.e $\tilde{\phi}(k) \geq 0$. In this regard we have :

$$f(k) - \frac{1+n}{p}k \geq \left(1 - \frac{R^*}{1+n}\right)\sigma$$

As long as the foreign rate of return on capital invested abroad R^* matches the following condition :

$$R^* < 1+n,$$

the capital mobility must verify the following condition for the net domestic production to be positive :

$$0 \leq \sigma \leq \left(1 - \frac{R^*}{1+n}\right)^{-1} \left(f(k) - \frac{1+n}{p}k\right). \quad (\text{B.84})$$

The previous condition specifies the limit of the level of capital individuals born in country 1 can invest abroad. In addition, one can deduce that at the steady-state γ -equilibrium that individuals in country 1 invest abroad if the steady-state rate of return on domestic savings is lower than the interest rate on capital invested abroad, which is lower than the rate of population growth. Given (B.28), it is straightforward to obtain :

$$pf'(k) < R' < 1+n \quad (\text{B.85})$$

It is important to note that the previous condition indicates an over-accumulation of capital in country 1 where the preference for quantity highly active at the first period of individuals' life-cycle (De la Croix and Michel, 2002).

The Golden rule k^{GR} is obtained by the following expression :

$$u'(\underline{c}^{GR} + \gamma\Delta^{GR}) = \frac{\beta}{1-\gamma} \left(\frac{1-\mu}{R} + \mu\right)^{-1} u'(d^{GR}) \quad (\text{B.86})$$

In this present chapter, the Golden rule k^{GR} is the stock of capital which corresponds to the stock of capital that is compatible with the stock of unsold goods and liquidity constraint. Furthermore, it is in accordance with the condition of capital mobility (B.84) and maximizes the net domestic production (B.81).

B.5 Monetary policy and Capital Mobility

Relaxing Assumption B.87, the aim of this section is to assess the impact of the monetary policy on the dynamics of capital when the preference for quantity principle is acting. Assuming that the government decides to create money. Denoting by λ_t the money growth rate during the period t , the total stock of money available in the economy is given as in Crettez et. al (1999) by :

$$\bar{M}_t = (1 + \lambda_t)\bar{M}_{t-1}$$

The parameter λ_t represents the rate of increase in the quantity of money used by the government to finance its governmental expenditures in the economy. Note this policy may hold individuals to increase consumption or savings. Since the preference for quantity principle traps the economy with stock of unsold goods for which production exceeds sales, savings of each individual must increase in order to allow a higher stock of capital per worker. The quantity of money created in the economy is given by :

$$\lambda_t \bar{M}_{t-1} = \bar{M}_t - \bar{M}_{t-1}$$

The latter allows the government to distribute a lump-sum transfer T_t^y to each young and a lump-sum transfer T_t^o to each old. Given above the government faces a budget constraint given by the following relation :

$$\lambda_t \bar{M}_{t-1} = N_t T_t^y + N_{t-1} T_t^o \tag{B.87}$$

In real terms, the first and second period constraints of each individual born in period t are respectively given by :

$$\underline{c}_t + s_t + \frac{M_t}{p_t} = w_t + \frac{T_t^y}{p_t}, \quad (\text{B.88})$$

$$d_{t+1} = R_{t+1}^r(s_t - \sigma_t) + \frac{R_{t+1}^* p_t}{p_{t+1}} \sigma_t + \frac{M_t}{p_t} + \frac{T_t^o}{p_{t+1}}. \quad (\text{B.89})$$

Isolating the saving from both the first and second period constraints and using Assumption B.7 related to the cash in advance constraint, we obtain the inter-temporal budget constraint with lump-sum transfers to individuals :

$$\underline{c}_t + \frac{1}{\vartheta_{t+1}} d_{t+1} = w_t - \nu_{t+1} \sigma_t + \frac{T_t^y}{p_t} + \frac{T_t^o}{p_{t+1}}, \quad (\text{B.90})$$

where ϑ_{t+1} is the total factor of anticipated rate of return on money and capital. Note that if the domestic and foreign rates of return on capital are equal, the factor ν_{t+1} vanishes. That does not mean that there is no mobility of capital across countries. However since the gap of rates of return on domestic and foreign capital is supposed to be positive, individuals born in country 1 have incentives to export their capital.

B.5.1 Consumption and Savings

Given the money creation by the government, each individual born in period t is endowed with a total amount of $w_t + \frac{T_t^y}{p_t}$ when young. In the second period of his lifetime, the consumption depends on the lump-sum transfer from government, his savings invested domestically or abroad and the money holding during the first period. Assuming that his inter-temporal utility is unchanged, the new maximization problem that each individual faces in period t is given by the following :

$$\mathcal{P}_2 : \left\{ \begin{array}{l} \text{Maximize} \quad u(\gamma(\bar{c}_t - \underline{c}_t) + \underline{c}_t) + \beta u(d_{t+1}) \\ \text{w.r.t.} \quad (\underline{c}_t, d_{t+1}) \in \mathcal{D}^\circ(U)_{\bar{c}, \cdot, d, \gamma} \times \mathcal{D}^\circ(U)_{\bar{c}, \underline{c}, \cdot, \gamma} \\ \text{s.t.} \quad \underline{c}_t + \frac{1}{\vartheta_{t+1}} d_{t+1} + \nu_{t+1} \sigma_t = w_t + \frac{T_t^y}{p_t} + \frac{T_t^o}{p_{t+1} R_{t+1}}, \\ \text{with} \quad \vartheta_{t+1} = \left(\frac{1-\mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-1} \quad \text{and} \quad \nu_{t+1} = \frac{1}{R_{t+1}^r} \left(R_{t+1}^r - \frac{R_{t+1}^* p_t}{p_{t+1}} \right) \\ \underline{c}_t \geq 0, d_{t+1} \geq 0. \end{array} \right. \quad (\text{B.91})$$

Since Assumptions 10 and 11 are unchanged, the first order condition of the new individual's maximization problem allows us to obtain the following Euler equation :

$$\frac{u'(\gamma(\bar{c}_t - c_t) + c_t)}{u'(d_{t+1})} = \frac{\beta}{1 - \gamma} \vartheta_{t+1} \quad (\text{B.92})$$

The first and second period constraints and the Euler relation (B.92) allow us to find the consumption and savings functions of the inter-temporal maximization problem of individuals born in period t . In the present paper if the parameter of preference for quantity $\gamma = 0$ the Euler equation is equivalent to the traditional relation between the intertemporal marginal rate of substitution (IMRS) and the total rate of return on saving invested abroad and the money held by individuals to consume in the second period. Thus, the first period consumption is optimally given by :

$$c_t := \underline{c}(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma). \quad (\text{B.93})$$

Replacing the first period consumption (B.93) into the first period constraint (B.88) provides us with the savings function as followings :

$$s_t = w_t + \frac{T_t^y}{p_t} - \underline{c}(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma) - \frac{M_t}{p_t},$$

which is defined by :

$$s_t := s(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma) \quad (\text{B.94})$$

Given the rate of return on capital invested abroad, the savings function of individuals born in period t is increasing in the amount of lump-sum transfer realized by the government. However, through the monetary policy, the impact of the government decision on the first period consumption may be more important.

Replacing the savings (B.94) into the second period constraint (B.89), we obtain the second period consumption :

$$d_{t+1} = R_{t+1}^r s(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma) - R_{t+1}^r \nu_{t+1} \sigma_t + \frac{M_t}{p_{t+1}} + \frac{T_t^o}{p_{t+1}} \quad (\text{B.95})$$

with $R_{t+1}^r \nu_{t+1} = \left(R_{t+1}^r - \frac{R_{t+1}^* p_t}{p_{t+1}} \right)$ and denoting by d the second period consumption, we have :

$$d_{t+1} := d(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma) \quad (\text{B.96})$$

Given the cash in advance constraint (B.22) and the anticipated second period consumption (B.96) the demand for money in period t is implicitly given by the following relation :

$$\frac{M_t}{p_t} = \mu \frac{p_{t+1}}{p_t} d(\bar{c}_t, w_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, R_{t+1}, R_{t+1}^*, T_{t+1}^o, \mu, \gamma). \quad (\text{B.97})$$

B.5.2 Capital Dynamics with Money Creation and Stock of Unsold Goods

In this subsection, each firm is characterized by the same production function defined in (B.27). The stock of capital in period t depreciates completely at the end of the period. Thus, the stock of capital in the next period results from current savings, which depends on the money creation and mobility of capital across countries. Furthermore, according to the remark 2 and given equations (B.28) and (B.29) related to the firm maximization problem, the dynamics of capital is given by the following :

$$k_{t+1} = \frac{1}{1+n} s(\bar{c}_t, f_t'(k_t) - f_t'(k_t)k_t, T_t^y, \sigma_t, M_t, p_t, p_{t+1}, p_{t+1}f'(k_{t+1}), R_{t+1}^*, T_{t+1}^o, \mu, \gamma) \quad (\text{B.98})$$

Implicitly, equation (B.98) defines the stock of capital in period $t+1$ as a function of the stock of capital in period t . It also allows us to determine the impact of the monetary policy stated by the government on the dynamics of capital in the open economy with stock of goods. However, it is important to account for this dynamics relatively to the preference for quantity generating a stock of unsold goods, the rate of increase in the stock of money λ_t and the way the government distributes the related lump-sum transfers between both young and old in period t and the rate of return on capital invested abroad.

B.5.2.1 Steady-state γ -equilibrium with Money Creation

The steady-state γ -equilibrium of the overlapping economy with money creation by the government can be rewritten as :

$$(1 + n)k = s - \sigma, \quad (\text{B.99})$$

Using the second period consumption (B.96) and the cash in advance constraint (B.8) that individuals face, we have :

$$m = \mu \left[Rs - R\nu\sigma + m + \frac{T^o}{p} \right],$$

which implies :

$$\frac{(1 - \mu)m}{\mu R} = s - \nu\sigma + \frac{T^o}{pR}. \quad (\text{B.100})$$

Isolating the savings from (B.99) and (B.101) and using the fact that the factor of interest rates gap $\nu = 1 - \frac{R^*}{R}$, the real steady-state quantity of money is given by :

$$\tilde{m} = \frac{\mu}{1 - \mu} \left[(1 + n)R\tilde{k} + R^*\sigma + \frac{T^o}{p} \right]. \quad (\text{B.101})$$

where \tilde{k} is the steady-state stock of capital in country that exports capital. The total money of steady-state γ -equilibrium depends on the level of lump-sum transfer to old, the return on capital invested abroad and the stock of capital used by the domestic producing firm.

B.6 Conclusion

Using a two-country overlapping-generations model, this paper have investigated the effects of the stock of capital on the steady-state welfare under the preference for quantity principle. Since the latter traps the economy stock of unsold goods, the stability condition of steady-state γ -equilibrium has changed the dynamics of capital. Indeed, under the cash-in-advance constraint imposed to consumer, he chooses the quantity of money compatible

with the level of savings invested abroad. Hence, the domestic producing firm imports one part of capital required to produce the quantity of goods allowing consumers to buy the quantity of first period consumption good. By construction, the model has the advantage of encompassing the neoclassical equilibrium for which the stock of unsold goods $\Delta_t = 0$ in period t as a sub-case.

Second, this paper has analyzed the steady-state welfare implications of stock of capital and mobility of capital in an open economy. Theorem 4 gives conditions under which the change in the inter-temporal indirect utility of individuals born in country 1 stemmed from the change in the total of capital abroad for given anticipated rates return and relative price between the first and the second period.

Otherwise, optimal stationary paths with stock of unsold goods and liquidity constraint were investigated. Based on the resource constraint and the highest stationary utility of individuals born in country 1 this chapter has showed that at the steady-state γ -equilibrium individuals invest their savings abroad if the domestic rate of return is lower than the foreign rate of return, which is in turn lower than the rate of population growth. The results have showed that the domestic economy is characterized by an over-accumulation of capital compatible with the preference for quantity which is active at the first period of individuals life-cycle.

The paper also analyzes the effect of the monetary policy on the economy. Given the rate of money creation and the weight of redistribution of lump-sum transfer of money between young and old, the monetary policy can have significant impact on the patterns of growth as well as on the rate of long-term growth of the economy. Assuming that the rate of money increase is variable, the model has determined the effect the monetary policy on the first and second period consumption. A policy based on the distribution of lump-sum transfer to young and old allows an optimal path of money held by individuals.

B.7 Appendix

B.7.1 Examples of a CIES function

Consider a two-country OLG with constant inter-temporal elasticity of substitution (CIES) utility function and a Cobb-Douglas production. The CIES-utility function is given by :

$$U(\bar{c}_t, c_t, d_{t+1}) = \frac{1}{1 - \frac{1}{\sigma}} (\gamma(\bar{c}_t - c_t) + c_t)^{1 - \frac{1}{\sigma}} + \frac{\beta}{1 - \frac{1}{\sigma}} d_{t+1}^{1 - \frac{1}{\sigma}}.$$

The young individuals solve the following inter-temporal maximizing problem :

$$\mathcal{P}_{CIES} : \begin{cases} \text{Maximize} & \frac{1}{1 - \frac{1}{\sigma}} (\gamma(\bar{c}_t - c_t) + c_t)^{1 - \frac{1}{\sigma}} + \frac{\beta}{1 - \frac{1}{\sigma}} d_{t+1}^{1 - \frac{1}{\sigma}} \\ \text{w.r.t.} & (c_t, d_{t+1}) \in \mathcal{D}^\circ(U)_{\bar{c}_t, d, \gamma} \times \mathcal{D}^\circ(\bar{U})_{\bar{c}_t, c, \gamma} \\ \text{s.t.} & c_t + s_t + \frac{M_t}{p_t} = w_t \\ & d_{t+1} = \frac{R_{t+1}}{p_{t+1}} (S_t - \Sigma_t) + \frac{R_{t+1}^*}{p_{t+1}} \Sigma_t + \frac{M_t}{p_{t+1}} \end{cases} \quad (\text{B.102})$$

The first order condition gives :

$$(1 - \gamma) [\gamma(\bar{c}_t - c_t) + c_t]^{-\frac{1}{\sigma}} = \beta \vartheta_{t+1} \left[\left(w_t - c_t - \frac{\sigma_t}{R_{t+1}^{a,r}} (R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}}) \right) \vartheta_{t+1} \right]^{-\frac{1}{\sigma}} \quad (\text{B.103})$$

where $\vartheta_{t+1} = \left(\frac{1 - \mu}{R_{t+1}^a} + \mu \frac{p_{t+1}}{p_t} \right)^{-1}$. The first period consumption of goods is given by :

$$c_t = \frac{\left(\frac{1 - \gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1 - \sigma} \left[w_t - \frac{\sigma_t}{R_{t+1}^{a,r}} (R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}}) \right] - \sigma \bar{c}_t}{1 - \gamma + \left(\frac{1 - \gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1 - \sigma}}. \quad (\text{B.104})$$

Given the first period budget constraint, the savings function is expressed as follows :

$$s_t = \frac{(1 - \gamma) w_t + \left(\frac{1 - \gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1 - \sigma} \frac{\sigma_t}{R_{t+1}^{a,r}} (R_{t+1}^{a,r} - R_{t+1}^* \frac{p_t}{p_{t+1}}) - (1 - \gamma + \left(\frac{1 - \gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1 - \sigma}) m_t + \gamma \bar{c}_t}{1 - \gamma + \left(\frac{1 - \gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1 - \sigma}} \quad (\text{B.105})$$

By the second period budget constraint, we have :

$$\underline{d}_{t+1} = \frac{(1 - \gamma) \left[R_{t+1}^{a,r} w_t + \sigma_t (R_{t+1}^* \frac{p_t}{p_{t+1}} - R_{t+1}^{a,r}) \right] + R_{t+1}^{a,r} \gamma \bar{c}_t}{1 - \gamma + \left(\frac{1-\gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1-\sigma}} + \left(\frac{p_t}{p_{t+1}} - R_{t+1}^{a,r} \right) m_t \quad (\text{B.106})$$

The demand for money is given by the following :

$$m_t = \frac{\frac{\mu(1-\gamma)}{1-\mu+R_{t+1}^a} \left[R_{t+1}^a w_t + \sigma_t (R_{t+1}^* - R_{t+1}^a) \right] + \frac{\mu\gamma}{1-\mu+R_{t+1}^a} R_{t+1}^a \bar{c}_t}{1 - \gamma + \left(\frac{1-\gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1-\sigma}}.$$

which can be written as :

$$m_t = \frac{\mu}{1 - \mu + R_{t+1}^a} \frac{(1 - \gamma) \left[R_{t+1}^a w_t + \sigma_t (R_{t+1}^* - R_{t+1}^a) \right] + \gamma R_{t+1}^a \bar{c}_t}{1 - \gamma + \left(\frac{1-\gamma}{\beta} \right)^\sigma \vartheta_{t+1}^{1-\sigma}}. \quad (\text{B.107})$$

B.8 Conclusion

Using the main results obtained from Blot, Gaumont and Cayemitte (2011), the goal of this part was twofold. On the one hand it has consisted in fitting the theoretical results based on CES and Cobb-Douglas functions to the quarterly USA data. In this regard, econometric considerations were used in order to highlight implications of consumers' behavior on both the level of production and inventory. On the other hand, an infinitely lived two-country overlapping generations model brought out the effects of the capital mobility and the monetary policy on the steady-state welfare.

In the chapter one of the present part, we have investigated the role of the preference for quantity for the inventory and economic fluctuations. In this respect, our modeling stemmed from the results of the theoretical model based particularly on perfect information and flexible price to the U.S. data from the first quarter of 1995 to the their quarter of 2011.

The nonlinearity of our model allows us to use a specific estimation method based on the Gauss-Newton algorithm. The results have showed that the parameter of the preference for quantity has a significant and positive impact on the inventory investment standed for the stock of unsold goods. They also have confirmed regularities that the production exceed sales, the production is more volatile than sales.

The conjoint evolution of change in inventory (CIP) and the GDP growth rate was analyzed. It has pointed out contrasted evolution of the real sales with respect to the inventory-to-sales ratio. As mentioned previously the positivity of this indicator foresees a bad sign for the economy. This is case of the U.S. economy, where it attained the highest level in 2008 related to the subprime crisis.

Furthermore by using a two-country overlapping-generations model, we have investigated in this document the effects of the stock of capital mobility on the steady-state welfare under the principle of preference for quantity and assumptions related to microeconomic literature.

We have analyzed the steady-state welfare implications of stock of capital and mobility of capital in an open economy. The last chapter has provided conditions under which the change in the inter-temporal indirect utility of individuals born in country 1 stemmed from the change in the total of capital abroad for given anticipated rates return and relative price between the first and the second period.

The effect of the monetary policy on the economy was also exhibited in the context of international capital mobility and the stock of unsold goods. Assuming that the rate of money increase is variable, the model has determined the effect the monetary policy on the first and second period consumption. According to the government policy, the savings may allow the producing firm to import less capital. This condition depends on the preference for quantity and the anticipated return on capital and money balances.

Conclusion

"I have never been able to grasp how one can understand any idea without knowing where it came from, how it evolved out of previous ideas ... Great theories, in economics as in other subjects, are path dependent ... that is, it is not possible to explain their occurrence without considering the corpus of received ideas which led to the development of that particular new theory; had the body of received ideas been different we would have arrived at a different theory at the culmination of that development. In other words, without the history of economics, economic theories just drop from the sky; you have to take them on faith. The moment you wish to judge a theory, you have to ask how they came to be produced in the first place and that is a question that can only be answered by the history of ideas". (Blaug, 1994)

Au terme de cette thèse, le moment est venu de faire un bilan des deux parties grandes élaborées. La première partie est constituée de deux chapitres : le premier s'est axé sur l'existence de stock d'inventures dans une structure de marché purement concurrentiel alors que le deuxième chapitre a mis l'accent sur l'existence de stock dans une structure de marché monopolistique avec notamment la déduction d'un indice de Lerner modifié, lequel permet la détermination du pouvoir de marché du monopole. La deuxième partie de la thèse était également divisée en deux chapitres. Le premier chapitre avait pour objectif d'analyser l'aspect empirique des modèles théoriques élaborés dans la première partie alors que le deuxième chapitre a consisté à analyser la question de préférence des individus dans le cadre d'une économie à Générations imbriquées.

En fin de compte, cette thèse a été développée selon trois principaux axes. Le premier axe a eu une portée purement théorique au cas de concurrence pure et parfaite et de monopole pure alors que le deuxième s'est intéressé à l'aspect empirique de la question relative au comportement des consommateurs et des producteurs. Enfin, le dernier s'est donné pour objectif d'analyser l'aspect relatif à l'accumulation et à la mobilité du capital dans une économie monétaire à générations imbriquées avec l'existence des stocks de biens.

B.9 Aspect théorique

Le premier axe de la thèse a montré théoriquement dans une économie néoclassique avec information parfaite, flexibilité des prix, absence des coûts d'ajustement, rationalité parfaite et absence de rationalité limitée qu'il existe un équilibre compatible avec le stock d'inventures. Il a été montré que ce nouveau type d'équilibre, appelé le γ -équilibre, contient

l'équilibre traditionnel néoclassique pour lequel il existe un prix qui égalise l'offre et la demande tant sur le marché concurrentiel que monopolistique.

B.9.1 Existence de stock de biens dans une structure de marché concurrentiel

Le premier chapitre de la thèse a proposé d'étudier l'existence du stock d'invendus dans une structure de marché purement concurrentiel. Cette question relative au stock de biens restant à la fin de la saison de ventes d'une entreprise est d'autant plus importante que la plupart des entreprises ou branches d'activités finissent leur saison de ventes avec des invendus ou au pire des stocks de biens invendables. Pour traiter de façon théorique cette question, nous avons considéré comme acquises les hypothèses néoclassiques telles : la flexibilité des prix, l'information parfaite, l'absence des coûts d'ajustement, la parfaite rationalité ou l'absence de rationalité limitée. Partant de ces hypothèses, il est traditionnellement admis d'après les néoclassiques qu'il existe un prix d'équilibre optimal pour lequel le stock d'invendus s'annule. Dans ce contexte économique, il n'est pas possible de parler de l'existence d'invendus.

Pour aborder théoriquement cette question, nous avons utilisé le *principe de préférence pour la quantité*. D'après ce principe, dans une structure de marché concurrentiel le consommateur est prêt à acheter un produit quelconque auprès du producteur s'il existe une quantité supérieure à ce dont il a besoin. Par exemple, pour acheter deux pommes dans le supermarché, il faut qu'il en y ait au moins trois pommes. Ce principe a permis de revoir le comportement traditionnel du consommateur représentatif et celui de la firme concurrentielle.

Étant donné le principe de préférence pour la quantité et les hypothèses néoclassiques traditionnelles de base, la résolution du modèle théorique a permis d'aboutir à trois résultats principaux pour le stock de biens invendus. Le premier résultat indique que si le consommateur a une contrainte budgétaire alors la firme concurrentielle peut faire un don ou donner gratuitement son stock d'invendus particulièrement aux organisations caritatives. Le deuxième nous dit que si le consommateur fait face à une contrainte budgétaire,

alors il peut écouler le stock d'invendus au prix du γ -équilibre, c'est-à-dire le prix d'équilibre compatible avec les stocks d'invendus. Cependant, ce prix n'est pas optimal pour la firme concurrentielle puisqu'il est le prix de marge réduite. Enfin, le troisième cas est celui où le consommateur n'a aucune contrainte budgétaire. Pour cela, la firme concurrentielle est incitée à détruire ou recycler son stock d'invendus.

Puisque la préférence pour la quantité tire l'économie vers une situation de γ -équilibre, aucun gouvernement ne peut intervenir durant la période de vente pour l'influencer directement, car rien ne peut faire contre les préférences des individus. Toutefois, nous avons relaté la possibilité au gouvernement d'intervenir sur les marchés du déstockage (marché induit par l'existence des invendus) pour une organisation efficace. Dans ce contexte, il peut créer des incitations permettant aux acteurs de s'organiser sur ce type de marché de façon optimale. Mais, ceci est loin d'être le cas dans la réalité. Comme le prix optimal sur le marché du déstockage ne permet pas d'équilibrer le marché sur lequel les préférences pour la quantité sont supposées exister et que dans certains cas ce prix existe, il y a une possibilité pour que le gouvernement agisse comme un déstockeur, ce qui permet, sur le plan macroéconomique de diminuer les pertes en termes de valeurs ajoutées pour l'économie, donc de limiter la perte de l'emploi.

B.9.2 Existence de stock de biens dans une structure de marché monopolistique

Le deuxième chapitre de la première partie de la thèse avait pour objectif d'exhiber le rôle du principe de préférence pour la quantité dans l'émergence du γ -équilibre dans une structure de marché monopolistique. Dans ce contexte de flexibilité des prix, où l'abondance des biens en termes de quantité étalée procurent au consommateur un certain nombre de services, l'équilibre de marché monopolistique est caractérisé par des stocks d'invendus à la fin de la période de vente. Ceci paraît intéressant dans la mesure où les stocks sont déterminés de façon optimale puisqu'ils ne sont dus à aucune défaillance du marché.

Dans un marché sans aucune friction, les résultats de notre modèle nous permettent de mettre en évidence les régularités suivantes selon lesquelles la production dépasse les

ventes, la production est plus volatile que les ventes mais ces dernières évoluent au même rythme au voisinage du γ -équilibre. Comme ça a été le cas du marché concurrentiel, les stocks d'invendus ou invendables se trouvant sur le marché du déstockage à la suite de la saison de ventes du monopoleur, peuvent être soit utilisés comme des dons, soit détruits ou recyclés.

Dans notre modèle théorique développé, le monopoleur joue à la fois sur le prix et la quantité \bar{q} afin d'attirer la demande des consommateurs. Comme \bar{q} est une variable stratégique, une extension naturelle de l'indice de Lerner a été proposée en prenant en compte la production d'une quantité \bar{q} suffisamment élevée relativement à la vraie quantité de biens achetée par les consommateurs. Cet indice mesurant le pouvoir de marché du monopole est défini par le rapport de la différence entre le prix d'un bien et son coût marginal sur le prix de ce bien. Mais ce coût marginal est consécutif à la quantité de biens générée par la présence du principe de préférence des consommateurs. Tant que ces deux niveaux de demande de biens sont décroissants par rapport au prix, la théorie avec des stocks de biens à l'équilibre soutenue dans cette thèse est considérée comme une extension de la théorie économique néoclassique traditionnelle.

Il est généralement admis que la concurrence pure et parfaite est plus favorable au bien-être des individus que le monopole. Cependant, si la demande de biens \bar{q} croît au prix, et ce qui est démontré dans notre modèle, alors l'équilibre concurrentiel, quoique produise plus de biens que le monopole, peut donner lieu à des prix plus élevés que ceux du monopole. La quantité de biens \bar{q} se sert alors des coûts fixes de telle sorte qu'un grand nombre de firmes, en étalant moins de biens que le fasse en petit nombre, puisse être plus efficace, dans la mesure où les biens sont fixés en deçà du prix du γ -équilibre concurrentiel.

B.10 Aspect empirique

L'objectif principal du troisième chapitre était d'adapter le modèle théorique aux données observées de l'économie américaine sur la période allant du premier trimestre 1995 au troisième trimestre 2011. Ceci nous a permis de quantifier le paramètre relatif au principe

de préférence pour la quantité et d'analyser les implications du nouveau comportement des consommateurs (et plus largement celui des ménages) pour les fluctuations économiques.

B.10.1 Stock de Biens invendus : Cause des Fluctuations économiques

Afin de vérifier la robustesse de notre modèle théorique, nous avons utilisé la méthode de régression de Gauss-Newton. Le choix de cette méthode est basé sur le fait que le modèle économétrique obtenu à partir des résultats de notre modèle théorique est non-linéaire par rapport aux paramètres. Les résultats des modèles théoriques ont été déterminés par l'application des fonctions d'utilité Log et CES puisque ces deux types de fonctions présentent des résultats différents, mais intéressants pour l'analyse de l'évolution de la production et des stocks de biens.

Les résultats empiriques de l'estimation des modèles économétriques ont indiqué un effet significatif du paramètre de préférence pour la quantité tant sur les stocks de biens que la production. Son effet sur la production de biens non durables est positive et est évalué à environ 0,61. En outre, ce paramètre est statistiquement significatif au seuil de 0,05. La valeur de ce coefficient montre l'importance de la préférence des consommateurs sur le stock de biens, par conséquent sur la valeur ajoutée. En matière de prévisions, un tel indicateur sera très pertinent pour l'explication des fluctuations économiques.

Nous avons également analysé dans ce chapitre l'évolution conjointe de la variation des stocks CIPI (change in private inventories or change in inventories investment) et du taux de croissance du produit intérieur brut (PIB) des Etats-Unis. Nous avons pu relever une évolution contrastée entre les ventes réelles et l'écart entre les stocks de biens et les ventes. Cet écart (ou ratio stocks de biens-ventes) est considéré comme un bon indicateur pour la prévision des fluctuations économiques. Une valeur positive laisse présager un mauvais signe pour l'économie. C'est le cas par exemple de l'économie américaine, où cet indicateur a enregistré le niveau le plus élevé en 2008, à la suite de la crise des subprimes qui a touche le secteur des prêts hypothécaires à risque aux États-Unis à partir de juillet 2007. En outre, les résultats fournis par la régression de Gauss-Newton (GNR) sont conformes

à notre analyse théorique selon laquelle la production est plus volatile que les ventes. En effet, si les ventes réelles augmentent d'environ un pour cent alors la production de biens durables augmente d'environ 3 pour cent.

B.11 Stock de biens dans une économie monétaire à Générations imbriquées

Dans un modèle à générations imbriquées à deux pays, le dernier chapitre avait pour objectif d'étudier les effets du stock de capital sur le bien-être à l'état stationnaire en présence du principe de préférence pour la quantité et de la création monétaire. Aussi avons-nous donné les conditions dans lesquelles l'état stationnaire du γ équilibre en autarcie par rapport à la contrainte sur la détention préalable d'encaisse est localement stable. Toutefois, dans un système caractérisé par la mobilité internationale des capitaux, la quantité de monnaie choisie est compatible avec le niveau de l'épargne investie à l'étranger. Par construction, le modèle a l'avantage d'englober comme un sous-cas, l'équilibre néoclassique pour lequel le stock d'inventus disparaît.

Selon le principe de préférence pour la quantité, nous avons montré que le stock d'inventus au γ d'équilibre est fonction de l'inflation et la demande de monnaie. Le fait que le stock d'inventus à la période t est positive a pour conséquence une hausse des prix et un excès d'offre de monnaie. S'il y a égalité entre l'offre de monnaie et la demande de monnaie à la période t alors le stock de biens inventus ne dépend que du capital de γ -équilibre pondéré par le taux d'inflation. A cet effet, étant donné le stock de capital et le taux de rendement domestique, pour diminuer le stock de marchandises inventus, il est nécessaire d'appliquer une politique en faveur de la baisse de l'inflation dans l'économie.

Étant donnés les taux de rendement domestiques et étrangers anticipés et le prix relatif de la consommation de la première par rapport à celle de la deuxième période, nous avons déterminé d'une variation du stock du capital investi à l'étranger sur l'utilité indirecte intertemporelle. En outre, étant donné le taux de croissance de la quantité totale de monétaire et le poids de la redistribution de transfert forfaitaire de la monnaie créée entre les jeunes

et les vieux, la politique monétaire peut avoir un impact significatif sur la trajectoire de croissance économique ainsi que sur le taux de croissance à long terme de la économie.

En supposant que le taux croissance de la quantité totale de monnaie dans l'économie domestique est variable, le modèle OLG avec le principe de préférence pour la quantité de biens a déterminé l'effet de la politique monétaire sur la consommation de la première et de la deuxième période de vie d'un jeune né dans la période t . En effet, pour atteindre son objectif par rapport à un modèle de croissance économique de long terme, le gouvernement peut créer plus de incitations afin de permettre aux jeunes nés dans le pays 1 au cours de la période t d'investir davantage. Par conséquent, compte tenu des taux de rendement anticipés sur le stock du capital investi, les agents peuvent allouer une part moins importante de leur salaire gagné au cours de la période t tant à l'investissement national et qu'à celui investi à l'étranger. Ce choix est basé sur le fait que les jeunes anticipent du gouvernement un transfert forfaitaire de la quantité totale de monnaie créée au cours de la première période.

Table des figures

1.1	Standard Equilibrium and γ -equilibrium : Case 1	63
1.2	Standard Equilibrium and γ -equilibrium : Case 2	64
1.3	Standard Equilibrium and γ -equilibrium : Case 3	64
1.4	Construction of the γ -equilibrium : Case 1	72
1.5	Construction of the γ -equilibrium : Case 2	73
1.6	Construction of the γ -equilibrium : Case 3	73
2.1	Increase of Monopoly Price Relative to γ	117
2.2	Increase of Monopoly Price Relative to γ and v_0	118
2.3	Increase of Monopoly Price Relative to γ and lower constant Value of v_0	119
2.4	Increase of Monopoly Price Relative to γ and lower Value of v_0	120
2.5	Monetary policy and the Phillips curve	127
A.1	Change in US private inventories	157
A.2	Change in inventory and Economic growth	160
A.3	Inventory to Sales ratio (I-S ratio)	161
A.4	Real Sales and Inventory to Sales ratio (I-S ratio)	162
A.5	GDP growth rate and Inventories to sales ratio from 1995 :1 to 2011 :3	171

Liste des tableaux

1.1	Corner and Interior Solutions for the Competitive Market Period	86
1.2	Results of Stock for the Selling-off Market Period	87
2.1	Corner and Interior Solutions for the Monopoly Market Period	123
2.2	Results of Stock of Unsold Goods for the Monopoly Selling-off Period . . .	123
A.1	NLLS estimation by the Gauss-Newton Method of the nonlinear regression (A.9)	164
A.2	NLLS estimation by the Gauss-Newton Method of the nonlinear regressions (A.11)	164
A.3	GNR based on the Restricted NLLS estimation of the nonlinear regressions (A.11)	165
A.4	Results of GNR Estimation of the stock of US Unsold goods	172
A.5	Results of GNR Estimation of the production	173
A.6	Results of GNR Estimation of the production (continued)	174
A.7	Results of the stock of goods (model with the Log utility function)	175

Bibliography

Bibliographie

- [1] Abramovitz, M. (1950), Inventories and Business Cycles, *National Bureau of Economic Research*, New York.
- [2] Allais, M. (1947) *Economie et Intérêt*, Imprimerie Nationale, Paris.
- [3] Allais, M., (1989) *La théorie générale des surplus*, PUG, Grenoble, France.
- [4] Archibald, Blyth C. (1981), "Continuous Review (s, S) Policies with Lost Sales", *Management Science*, 27 (10), pp. 1171-1177.
- [5] Arrow Kenneth J., G. Debreu (1954) "Existence of an Equilibrium for a Competitive Economy" *Econometrica*, 22(3) : 265-290.
- [6] Arrow Kenneth J., L. Hurwicz (1958) "On the Stability of the Competitive Equilibrium," *Econometrica* 26 : 522-552.
- [7] Baker, R.C. and Timothy L. Urban (1998) "A deterministic Inventory System with an Inventory-Level-Dependent Demand Rate" *The Journal of Operational Research* 39(9) : 823-831.
- [8] Balakrishnan A., M. S. Pangburn, E. Stavroulaki (2004) "‘Stack Them High, Let’em Fly’ : Lot-Sizing Policies When Inventories Stimulate the Demand", *Management Science*, 50(5) : 630-644.

- [9] Baltagi, Bati H. (2008), "*Econometrics*", 4th Ed. Syracuse University, Center for Policy Research, NY, USA.
- [10] Barro, R.J., and H.J. Crossman (1976), "*Money, Employment and Inflation*". Cambridge : Cambridge University Press.
- [11] Bates, D.M. and Watts, D.G. (1988). *Nonlinear Regression Analysis and Its Applications*, Wiley, New York.
- [12] Battberg, R.C. and S.A. Neslin (1990) *Sales, Promotion concepts, Methods and Strategies* Prentice Hall, Englewood cliffs, NJ.
- [13] Blinder, Alan S. (1981), "Inventories and the Structure of Macro Models," *American Economic Review*, LXXI, pp. 11-16.
- [14] Blinder, Alan S., Lovell, Michael C., Summers, Lawrence H. (1981), Retail Inventory Behavior and Business Fluctuations, Reviewed work(s) *Brookings Papers on Economic Activity*, 1981 (2), pp. 443-520.
- [15] Blot, Cayemitte and Gaumont (2011) "Why are there no Firms without Stocks of Unsold Goods? ", *ERMES Working papers, EAC 7181 CNRS*, 11-20.
- [16] Bonanno, G.(1990), General equilibrium theory with imperfect competition. *Journal of Economic Surveys* 4, 297-328
- [17] Borts, George H. (1964), A Theory of Long-Run International Capital Movements, *Journal of Political Economy*, 72 (4), 341-59.
- [18] Bryant, John, (1980) "Competitive Equilibrium with Price Setting Firms and Stochastic Demand," *International Economic Review* 21(4) : 619-626.

- [19] Buiter, Willem H. (1981), Time Preference and International Lending and Borrowing in an Overlapping-Generations, *Journal of Political Economy*, 89 (4), pp. 769-797
- [20] Bulow, J. (1982), "Durable Goods Monopolists", *Journal of Political Economy*, 90, 314-332.
- [21] Buse, A. (1982), "The Likelihood Ratio, Wald and Lagrange Multiplier Tests : An Expository Note", *The American Statistician*, 36 (3).
- [22] Buse, A. (1981), Pooling Times Series and Cross Section Data : A Minimum Chi-Squared Approach, *The Manchester School*, 49, 229-244
- [23] Carlton Dennis W. (1986) "The Rigidity of Prices" *The American Economic Review*, 76(4) : 637-658.
- [24] Cartan, H., *Calcul différentiel* Hermann, Paris, 1977.
- [25] Corstjens, M. and Peter Doyle (1981) "A Model for Optimizing Retail Space Allocation", *Management Science*, 27(7) : 822-833.
- [26] Corstjens, M. and Peter Doyle (1983) "A Dynamic Model for Strategically Allocating Retail Space", *Journal of the operational research society*, 34(10) :943-951.
- [27] Crettez, B., Michel, P., Wigniolle, B. (1998), Contraintes de Liquidités dans le Modèle à Générations Imbriquées de Diamond. Neutralité des Politiques Monétaires et Dynamique Macroéconomique, *Revue économique*, 49 (3), pp. 677-685.
- [28] Dana, James D. Jr (1993) "Equilibrium price dispersion under demand uncertainty : the role of Costly Capacity and Market Structure," *RAND Journal of Economics* 30(4) : 632-660.

- [29] Darling, Paul and Lovell, Michael C. (1971), Inventories, Production Smoothing, and the Flexible Accelerator, *The Quarterly Journal of Economics*, 85 (2), pp. 357-362.
- [30] De La Croix D. and Ph. Michel (2002) "*A Theory of Economic Growth Dynamics and Policy in Overlapping Generations*", Cambridge University Press.
- [31] Deneckere, R., H. P. Marvel, and J. Peck (1996) "Demand Uncertainty, Inventories, and Resale Price Maintenance" *The Quarterly Journal of Economics*, 111(3) : 885-913.
- [32] Diamond, P. (1965) National Debt in Neoclassical Growth Models, *The American Economic Review*, 55 : 1126-1150.
- [33] Dugundji, J., *Topology*, Allyn and Bacon, Inc., Boston, 1966.
- [34] Eden, Benjamin (1990) "Marginal Cost Pricing When Spot Markets Are Complete", *Journal of Political Economy*, 98 : 1293-1306.
- [35] Eckalbar, John, C. (1984), Inventories in a Dynamic Macro Model with Flexible Prices, *European Economic Review*, 27, pp. 201-219.
- [36] Enders, Walter (2003) "*RATS Programming Manuel*", Department of Economics, Finance and Legal Studies, University of Alabama.
- [37] Farmer, Karl and Wendner, Ronald (2003), A Two-Sector Overlapping Generations Model with Heterogeneous Capital, *Economic Theory*, 22 (4), pp. 773-792
- [38] Fischer, Stanley, and Frenkel, Jacob A. "Investment, the Two-Sector Model and Trade in Debt and Capital Goods, *Journal International Economic*, 211-33.

- [39] Florenzano Monique (1987), Equilibrium in a Production economy on an Infinite Dimensional Commodity Space. A Unifying Approach, *CEPREMAP Working Papers 8740, CEPREMAP*.
- [40] Galor, Oded (1986), Time Preference and International Labor Migration, *Journal of Economic Theory*, 38, pp. 1-20.
- [41] Galor, Oded (1992), A Two-Sector Overlapping-Generations Model : A Global Characterization of the Dynamical System, *Econometrica*, 60 (6), pp. 1351-1386
- [42] Gallant, A. R. (1975) "Nonlinear Regression", *The American Statistician*, 29 (2) : 73-81.
- [43] Ghali, Moheb (1974), *Inventories, Production Smoothing, and the Accelerator : Some Empirical Evidence*, *The Quarterly Journal of Economics*, 88(1) :149-157.
- [44] Gaumont D., M. Schindler and R. Wright (2006) "Alternative theories of wage dispersion" *European Economic Review*, 50(4) : 831-848.
- [45] Grandmont Jean-Michel (1975), "Temporary General Equilibrium Theory" *Econometrica*, 45 :535-572.
- [46] Greene, H., William (2003) "Interpreting Estimated Parameters and Measuring Individual Heterogeneity in Random Coefficient Models", *Working Papers 03-19, New York University, Leonard N. Stern School of Business, Department of Economics*.
- [47] Gul, F., H. Sonnenschein, and R. Wilson (1986) : "Foundations of Dynamic Monopoly and the Coase Conjecture", *Journal of Economic Theory*, 39, 155-190.
- [48] Gurnani H. and Z. Drezner (2000) "Deterministic Hierarchical Substitution Inventory Models" *The Journal of the Operational Research Society*, 51(1) :129-133.

- [49] Hart, O.D. : Imperfect competition in general equilibrium : An overview of recent work.
In : Arrow, K., Honkapohja, S. (eds.) *Frontiers of economics*, pp. 100-149. Oxford : Blackwell 1985.
- [50] Hay, George A. (1970), "Adjustment Costs and the Flexible Accelerator", *The Quarterly Journal of Economics*, LXXXIV, 14043.
- [51] Hayashi, Fumio (2000) "*Econometrics*", Princeton University Press, Oxford.
- [52] Holt, Charles C., Modigliani Franco, Muth John F., Simon Herbert A.(1962), Planning Production, Inventories, and Work Force, Review by G. S. Tolley *Journal of Farm Economics*, 44 (2), pp. 650-652.
- [53] Kawasaki Seiichi , John McMillan, Klaus F. Zimmermann (1983) "Inventories and Price Inflexibility" *Econometrica*, 51(3) : 599-610.
- [54] Khan Aubhik, and Julia K. Thomas (2007) "Inventories and the Business Cycle : An Equilibrium Analysis of (S, s) Policies Inventories and the Business Cycle : An Equilibrium Analysis of (S, s) Policies" *The American Economic Review*, 97(4) : 1165-1188.
- [55] Kahn James A., Margaret McConnell, and Gabriel Perez-Quiros (2002)"On the Causes of the Increased Stability of the US Economy" *Federal Reserve Bank of New York, Economic Policy Review* / May 2002, 183-202.
- [56] Kemp, Murray, C., Wong, Kar-yiu (1995), Gains from Trade with Overlapping Generations, *Economic Theory*, 6 (2), pp. 283-303
- [57] Kollat, David T. and Willett, Ronald P. (1967) "Customer Impulse Purchasing Behavior quick view", *Journal of Marketing Research*, 4(1) : 21-31.

- [58] Kydland, Finn E. and Prescott, Edward C. (1996) "The Computational Experiment : An Econometric Tool," *Journal of Economic Perspectives*, 10(1), pp. 69-85.
- [59] Lange, Oscar (1942), "Say's Law : A Restatement and Criticism." *In Studies in Mathematical Economics and Econometrics*, edited by Oscar Lange et al. (Chicago : University of Chicago Press), pp.49-68.
- [60] Larson, P.D., and P.A. DeMarais (1990) "Psychic Stock : An Independent Variable Category of Inventory". *International Journal of Physical Distribution Logistic Management*, 20 : 28-35.
- [61] Lazear, Edward P. (1986) "Retail Pricing and Clearance Sales" *The American Economic Review*, 76(1) : 14-32.
- [62] Levin , R. I., C.P. McLaughlin, R.P. Lamone and J.F. Kottas (1972) *Production / Operations Management : Contemporary Policy for Managing Operating Systems*. McGraw-Hill, New York.
- [63] Lerner, A. P., "The Concept of Monopoly and the Measurement of Monopoly Power", *The Review of Economic Studies*, (1934), 1(3) : 157-175.
- [64] Lovell, Michael C. (1961), "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle", *Econometrica*, 29 (3), pp. 293-314
- [65] Lovell, Michael C. (1962), "Buffer Stocks, Sales Expectations, and Stability : A Multi-sector Analysis of Inventory Cycle", *Econometrica*, 30 (2), pp. 267-296
- [66] Lucas, Robert E., and Michael Woodford, (1993) "Real Effects of Monetary Shocks in an Economy with Sequential Purchases," *NBER Working Paper* No. 4250, January 1993.

- [67] Lundberg, Erik (1955), *Studies in the Theory of Economic Expansion*, Kelly and Millman.
- [68] Maccini, Louis J., and Robert Rossana, "Investment in Finished Goods Inventories : An Analysis of Adjustment Speeds", *American Economic Review*, LXXI (1981),17-22.
- [69] Marquardt, Donald W. (1963) "An Algorithm for Least-Squares Estimation of Non-linear Parameters", *Journal of the Society for Industrial and Applied Mathematics*, 11 (2) : 431-441
- [70] Mathewson G.F., and R.A. Winter (1987) "The Competitive Effects of Vertical Agreements : comment" *American Economic Review* 77(5) :1057-1062.
- [71] Metzler, L. (1941), The nature and stability of inventory cycles, *Review of Economics and Statistics* 23, 113-129.
- [72] Miron, Jeffrey A., and Stephen Zeldes (1987) "Seasonality, Cost Shocks and the Production Smoothing Model of Inventories," *NBER Working Paper*, 2360.
- [73] Nerlove, M., 1963, "Returns to Scale in Electricity Supply", in C. Christ (ed.), *Measurement in Economics : Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, Stanford : Stanford University Press.
- [74] Pareto, Vilfredo, (1968), *The Rise and Fall of the Elites : an Application of Theoretical Sociology*, Totowa, N.J., Bedminster Press.
- [75] Prescott, Edward C., (1975) "Efficiency of the Natural Rate", *Journal of Political Economy*, 83 : 1229-1236.
- [76] Rand, Ayn (1957), *Atlas Shrugged*, Random House, USA.

- [77] Ratkowsky, D.A. (1983). *Nonlinear Regression Modelling : A Unified Practical Approach*, Marcel Dekker, New York.
- [78] Rey P., and Tirole, J., (1986) "Contraintes verticales : l'Approche principal-agent", *Annales d'économie et de Statistique*, 1 : 175-201.
- [79] Samuelson, P.A. (1958) "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money", *Journal of Political Economy*, Vol.66 : 467-482.
- [80] Scarf, Herbert E. (1960) "The Optimality of (S, s) Policies in the Dynamic Inventory Problem", in *Mathematical Methods in the Social Sciences*, ed. K. J. Arrow, S. Karlin, and H. Scarf, 196-202. Stanford : Stanford University Press.
- [81] Scarf, Herbert E. (2002) "Inventory Theory", *Operations Research*, 50(1) : 186-191.
- [82] Schutte, D. P., (1984) Optimal Inventories and Equilibrium Price Behavior, *Journal of Economic Theory*, 33 : 46-58.
- [83] Seber, G.A.F. and Wild, C.J. (1989). *Nonlinear Regression*, Wiley, New York.
- [84] Shaffer Greg (1991) "Capturing Strategic Rent : Full-Line Forcing, Brand Discount, Aggregate Debate and Maximum Resale Price Maintenance", *The Journal of Industrial Economics* 39(5) :567-575.
- [85] Smith, Adam (1776) *An Inquiry into the Nature and Causes of the Wealth of Nations*, 5th ed. (1904), London : Methuen & Co., Ltd.
- [86] Snowdon, Brian and Vane, Howard R. (2005) *Modern Macroeconomics : Its Origins, Development and Current State*, Edward Elgar (EE)
- [87] Silver, E.A. and R. Peterson (1985) *Decision systems for Inventory Management and production Planning*, John Wiley, New York.

- [88] Joel Sobel, (1991), Durable Goods Monopoly with Entry of New Consumers, *Econometrica*, 59 (5), pp. 1455-1485.
- [89] Uzawa Hirofumi, (1961) "Market Mechanisms and Mathematical Programming" *Econometrica*, 28(4) : 872-881.
- [90] Varian, H. R., *Microeconomic Analysis*, McGraw-Hill, Boston Massachusetts, 1992.
- [91] Vidal, J.-P. (2000), Capital Mobility in a Dynastic Framework, *Oxford Economic Papers*, 52 (2000), pp. 606-625.
- [92] Walras, Léon (1874) "*Element d'Economie Pure, ou théorie de la richesse sociale*", R. Pichon and R. Durant-Auzias Eds. Paris, R. Rouge libraire Ed, 1926.
- [93] West, Kenneth D. (1990), "The Sources of Fluctuations in Aggregate Inventories and GNP", *The Quarterly Journal of Economics*, 105 (4), pp. 939-971.
- [94] Within T.M. (1957) *The theory of Inventory Management*, Princeton university press, Princeton, NJ.
- [95] Wolfe H.B., (1968) "A Model for Control of Style of Merchandise", *Industrial Management Review*, 9 : 69-82.
- [96] Wooldridge, Jeffrey M. (2002), *Econometric Analysis of Cross Section and Panel Data*. Cambridge, MA : MIT Press.

Résumé : Cette thèse construit un modèle théorique qui renouvelle l'approche traditionnelle de l'équilibre du marché. En introduisant dans le paradigme néo-classique le principe de préférence pour la quantité, il génère de façon optimale des stocks dans un marché concurrentiel. Les résultats sont très importants, car ils expliquent à la fois l'émergence des invendus et l'existence de cycles économiques. En outre, il étudie le comportement optimal du monopole dont la puissance de marché dépend non seulement de la quantité de biens étalés, mais aussi de celle de biens achetés. Contrairement à l'hypothèse traditionnelle selon laquelle le monopoleur choisit le prix ou la quantité qui maximise son profit, il attire, via un indice de Lerner généralisé la demande à la fois par le prix et la quantité de biens exposés. Quelle que soit la structure du marché, le phénomène d'accumulation des stocks de biens apparaît dans l'économie. De plus, il a l'avantage d'expliquer explicitement les achats impulsifs non encore traités par la théorie économique. Pour vérifier la robustesse des résultats du modèle théorique, ils sont testés sur des données américaines. En raison de leur non-linéarité, la méthode de Gauss-Newton est appropriée pour analyser l'impact de la préférence pour la quantité sur la production et l'accumulation de biens, et par conséquent sur les prévisions de PIB. Enfin, cette thèse construit un modèle à générations imbriquées à deux pays qui étend l'équilibre dynamique à un gamma-équilibre dynamique sans friction. Sur la base de la contrainte de détention préalable d'encaisse, il ressort les conditions de sur-accumulation du capital et les conséquences de la mobilité du capital sur le bien-être dans un contexte d'accumulation du stock d'invendus.

Mots clés : Comportement Microéconomique, Comportement des entreprises, la théorie économique du Consommateur, Concurrence Parfaite, Monopole, Fluctuations du PIB, Régression non Linéaire, Méthode de Gauss-Newton, Stock d'Invendus, Détention préalable d'Encaisse, Modèle à Générations Imbriquées, Mobilité du Capital, Politique Monétaire.

Abstract : This thesis constructs a theoretical model that renews the traditional approach of the market equilibrium. By introducing into the neoclassical paradigm the principle of preference for quantity, it optimally generates inventories within a competitive market. The results are very important since they explain both the emergence of unsold goods and the existence of economic cycles. In addition, it studies the optimal behavior of a monopolist whose the market power depends not only on the quantity of displayed goods but also that of goods that the main consumer is willing to buy. Contrary to the traditional assumption that the monopolist chooses price or quantity that maximizes its profit, through a generalized Lerner index (GLI) it attracts customers' demand by both the price and the quantity of displayed goods. Whatever the market structure, the phenomenon of inventory accumulation appears in the economy. Furthermore, it has the advantage of explicitly explaining impulse purchases untreated by economics. To check the robustness of the results, the theoretical model is fitted to U.S. data. Due to its nonlinearity, the Gauss-Newton method is appropriate to highlight the impact of consumers' preference for quantity on production and accumulation of goods and consequently GDP forecast. Finally, this thesis builds a two-country overlapping generations (OLG) model which extends the dynamic OLG equilibrium to a frictionless dynamic OLG gamma-equilibrium. Based on the cash-in-advance constraint, it highlights the conditions of over-accumulation of capital and welfare implications of capital mobility in a context of accumulation of stock of unsold goods.

Keywords : Microeconomic Behavior, Firm Behavior, Economic Theory of the Consumer, Perfect Competition, Monopoly, GDP Fluctuations, Nonlinear Regression, Gauss-Newton Method, Stock of Goods, Cash-in-Advance, Overlapping Generations Model, Capital Mobility, Monetary policy.